MA448: K-theory and Solitons
Problem Set 4

Due at 10 am on Thursday, 19 February 2009.

1 Clutching Functions

Consider an open covering $\{V_j\}$ of X and a bundle $p : E \to X$ with trivializations $h_j : p^{-1}(V_j) \to V_j \times \mathbb{C}^n$ and clutching functions $f_{ij} = h_i h_j^{-1}$. Prove that the bundle E is trivial if and only if there is a set of maps $g_j : V_j \to GL_n(\mathbb{C})$ such that for any i and j on $V_i \cap V_j$ the clutching functions f_{ij} satisfy $f_{ij}(x) = g_i^{-1}(x) g_j(x)$.

2 Pullback

Consider an n-covering map $f_n : S^1 \to S^1; z \mapsto z^n$. If $M \to S^1$ is the Möbius bundle, what is its pullback $f_n^* M$?

3 Cones and Suspension

For $A \subset X$, prove that $(X \cup CA) \cup CX$ is homotopy equivalent to the suspension SA.

4 Simplest K-group

Describe the bundle that determines the generator of the reduced K-group

$$\tilde{K}_{\mathbb{C}}(S^0) = \mathbb{Z}.$$