MA448: K-theory and Solitons
Problem Set 3

Due at 10 am on Thursday, 12 February 2009.

1-4. Smash Product

Define the *smash product* $X \wedge Y$ of two topological spaces X and Y by

$$X \wedge Y = X \times Y / X \vee Y.$$

Prove that

1. $S^n \wedge S^m = S^{n+m}$.

2. S^n / S^m is homotopically equivalent to $S^n \vee S^{m+1}$.

3. $S^n \backslash S^m$ is homotopically equivalent to S^{n-m-1}.

4. for any topological space X we have $X \wedge S^1 = SX$, where SX is the *suspension* of X. I.e. $SX = X \times I / (X \times \{0\} \coprod X \times \{1\}$.

5. Künneth Formula

(Problem 7.1 in H. Sato.)

Use the integral homology of the real projective plane

$$H_0(\mathbb{R}P^2; \mathbb{Z}) \cong \mathbb{Z}, \ H_1(\mathbb{R}P^2; \mathbb{Z}) \cong \mathbb{Z}_2, \ H_2(\mathbb{R}P^2; \mathbb{Z}) = 0$$

to find the homology groups

$$H_*(\mathbb{R}P^2 \times \mathbb{R}P^2; \mathbb{Z}).$$