The non-Lorentzian holographic universe

Chris Blair (IFT UAM-CSIC)

The Holographic Universe, Leuven, June 5th 2025

Based on:

"Matrix theory reloaded: A BPS road to holography"
2410.3591 with Johannes Lahnsteiner, Niels Obers and Ziqi Yan

In the first half of the talk I will present my main messages

In the second, I will run through some of the details

Motivation: BPS decoupling limits and non-Lorentzian geometry

Unified perspective set out in [CB, Lahnsteiner, Obers, Yan 2311.10654] [Gomis, Yan 2023]

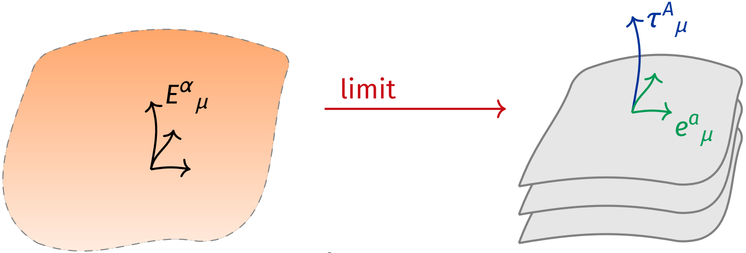
Goal: study string and M-theory in special limits ⟶ simpler subsectors

BPS decoupling limits: associated with BPS configuration [de Boer, Dijkgraaf, Harmark, Obers unpublished]

e.g. D-brane, string, M2, M5, intersections,...

Examples: AdS/CFT, matrix theory, non-relativistic string theory,...

Modern geometric perspective: BPS decoupling limits as non-relativistic limits



Lorentzian metric: $G_{\mu\nu} = E^\alpha{}_\mu E^\beta{}_\nu \eta_{\alpha\beta}$

Non-Lorentzian: metric description invalid

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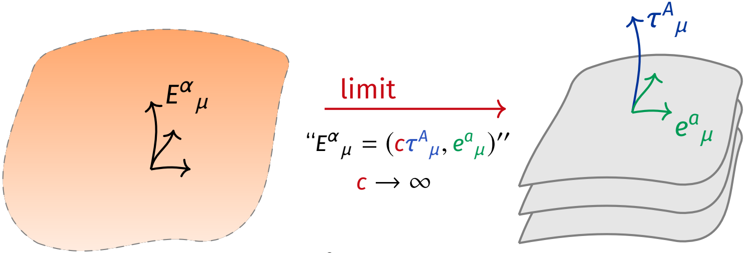

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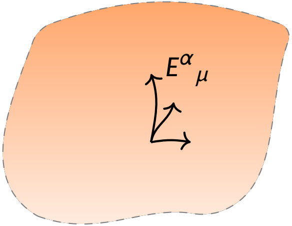
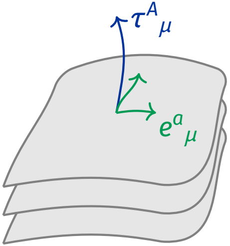


$\xrightarrow{\text{limit}}$

"$E^\alpha{}_\mu = (c\tau^A{}_\mu, e^a{}_\mu)$"

$c \longrightarrow \infty$



"pair of degenerate metrics"

$\tau_{\mu\nu} = \tau^A{}_\mu \tau^B{}_\nu \eta_{AB}$

$E_{\mu\nu} = e^a{}_\mu e^b{}_\nu \delta_{ab}$

Lorentzian metric: $G_{\mu\nu} = E^\alpha{}_\mu E^\beta{}_\nu \eta_{\alpha\beta}$

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Message 1: non-Lorentzian geometry underlies holography

[CB, Lahnsteiner, Obers, Yan 2410.3591]

The Dp-brane decoupling limit is a non-relativistic limit

generalises point particle non-relativistic limit D0 **is** point particle

Leads to SYM on stack of (aligned) Dp-branes n.b. caveats for $p > 3$

Same limit of curved Dp-brane geometry *transmutes* into the near-horizon limit

$$H = 1 + \left(\frac{L}{R}\right)^{\#} \longrightarrow H = \left(\frac{L}{R}\right)^{\#}$$

$R \longrightarrow \infty$: near-horizon geometry approaches an asymptotic flat (10-d) non-Lorentzian geometry $\longrightarrow$ same target space seen by dual field theory

more discussion [Guijosa 2025]

Other examples work similarly: M2, M5, intersecting branes [CB, Lahnsteiner, Obers, Yan 2410.3591], F1 [Harmark, Lahnsteiner, Obers 2025], duals of non-commutative YM [CB, Lahnsteiner, Obers, Yan 2502.20310]

Message 2: multiple decoupling limits give non-Lorentzian holography

Involving bulk geometries that do not have metric structure

Take a Dp limit of a Dq geometry consistent: 1/4 BPS configuration

$\Longrightarrow$ curved non-Lorentzian geometry, solution of non-Lorentzian SUGRA

Further Dq near-horizon limit: a non-Lorentzian holographic bulk

Proposed duals: limits of SYM which localise on BPS configurations [Lambert, Smith 2024 $\times 3$]

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Geometric perspective and conjectural taxonomy [CB, Lahnsteiner, Obers, Yan 2410.3591]

1 decoupling limit = in duality orbit of D0 non-rel limit $\longleftarrow$ DLCQ of M-theory

AdS/CFT as DLCQ^n/DLCQ^m $m > n$

offset: decoupling limits which induce near-horizon limits do not change nature of bulk geometry but always act on field theory

AdS$_5$/CFT$_4$: DLCQ^0/DLCQ^1 bulk near-horizon/asymptotic D3-brane non-Lorentzian limit

AdS$_3$/CFT$_2$: DLCQ^0/DLCQ^2 bulk near-horizon/asymptotic D1-D5-brane non-Lorentzian limit

Non-Lorentzian bulk examples: DLCQ^1/DLCQ^2 further limit of above

DLCQ^2/DLCQ^3? **Classify?**

Message 3: undoing decoupling limits gives field theory deformations

They generalise the $T\bar{T}$ deformation of 2d field theories

$T\bar{T}$ flow equation: $\frac{\partial \mathcal{L}(\lambda)}{\partial \lambda} \sim \det T_{\alpha\beta}(\lambda)$ [Zamolodchikov 2004; Smirnov, Zamolodchikov 2016]

[Cavaglià et al 2016; Bonelli et al 2018]

$$\boxed{\begin{array}{c} D \text{ free bosons} \\ \lambda = 0 \end{array}} \xrightarrow{T\bar{T}} \boxed{\begin{array}{c} \text{Nambu-Goto string in } D+2 \text{ dims} \\ \lambda \neq 0 \end{array}}$$

[CB 2002.12413]

$\lambda \to 0$

Non-Relativistic String Theory limit

Reverse brane decoupling limits: new $T\bar{T}$-like deformations [CB, Lahnsteiner, Obers, Yan 2410.3591]

Example in 3d: $$\frac{\partial \mathcal{L}(\lambda)}{\partial \lambda} = \frac{1}{4}\left(T_{\alpha\beta}T^{\alpha\beta} - (T_\alpha{}^\alpha)^2 - 2\lambda \det T_{\alpha\beta}\right)$$

n.b. so far this is all abelian, bosonic, classical

$$\boxed{\begin{array}{c} 7 \text{ free bosons + Maxwell field} \\ \lambda = 0 \end{array}} \xrightarrow{T\bar{T}\text{-like}} \boxed{\begin{array}{c} \text{DBI for D2-brane} \\ \lambda \neq 0 \end{array}}$$

$\lambda \to 0$

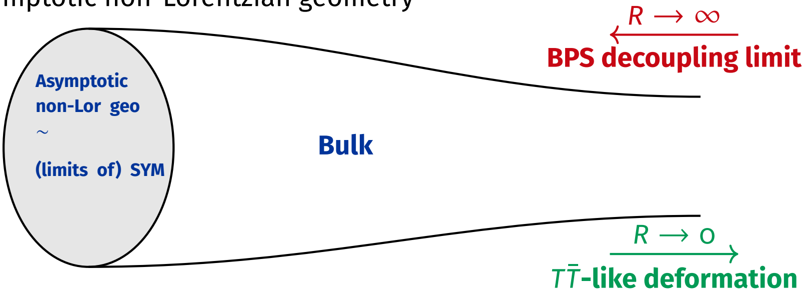
D2-brane decoupling limit

A surprising new geometric perspective on AdS/CFT?

1. **BPS decoupling limits** for particles, strings and branes are **non-relativistic limits**
2. The **AdS/CFT decoupling limit** is of this type

   non-Lorentzian geometry appears asymptotically
3. Applying multiple decoupling limits leads to a **non-Lorentzian holographic universe**

   waiting to be explored further
4. BPS decoupling limits **in reverse** give **$T\bar{T}$-like deformations** of field theories
5. Near-horizon brane geometries arise from **intrinsic geometric $T\bar{T}$-like deformations** of an asymptotic non-Lorentzian geometry



Now I will run through some details... in more detail

A non-relativistic particle limit of string theory

Using D0-branes and leading to matrix theory

$$X^\mu = (\omega^{1/2} t, \omega^{-1/2} x^a) \qquad \ell_s \text{ fixed}$$
$$G_s = \omega^{-3/2} g_s \qquad C_{(1)} = \omega^2 g_s^{-1} \mathrm{d}t$$
$$\mathrm{d}s^2 = -\omega \mathrm{d}t^2 + \omega^{-1} \mathrm{d}x^a \mathrm{d}x^a$$

c.f. non-relativistic limit of charged point particle

$S = -mc \int \mathrm{d}\tau \sqrt{c^2 \dot{t}^2 - \dot{x}^a \dot{x}^a} + q \int A_{(1)}$

BPS $m = q = 1/(g_s \ell_s)$ and tune to 'critical' gauge field

$$S_{\mathrm{D0}} = -\frac{1}{G_s \ell_s} \int \mathrm{d}\tau \sqrt{-\eta_{\mu\nu} \dot{X}^\mu \dot{X}^\nu} + \frac{1}{\ell_s} \int C_{(1)} = -\frac{\omega}{g_s \ell_s} \int \mathrm{d}\tau \sqrt{\omega^2 \dot{t}^2 - \dot{x}^a \dot{x}^a} + \frac{\omega^2}{g_s \ell_s} \int \mathrm{d}t$$
$$\xrightarrow{\omega \to \infty} \frac{1}{g_s \ell_s} \int \mathrm{d}t \frac{1}{2} \frac{dx^a}{dt} \frac{dx^a}{dt}$$

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M-theory uplift $R = g_s \ell_s$

$$\mathrm{d}s_{11}^2 = \omega^{-2} R^2 (\mathrm{d}x^{10})^2 + 2R \mathrm{d}x^{10} \mathrm{d}t + \mathrm{d}x^a \mathrm{d}x^a \qquad \ell_p \text{ fixed}$$

$\omega \to \infty$: null compactification (DLCQ) of M-theory $\longrightarrow$ matrix theory on D0-branes

Infinite boost of spatial compactification $\beta \approx 1 - \omega^{-2}$

[de Wit, Hoppe, Nicolai 1988]
[Banks, Fischler, Schenker, Susskind 1996]
[Susskind; Seiberg; Sen 1997]

More generally, BPS decoupling limit of a p-brane

Time/space split $\longrightarrow$ longitudinal/transverse split

$$X^\mu = (\omega^{1/2}x^A, \omega^{-1/2}x^a) \qquad \ell_s \text{ fixed}$$

$$A = 0, 1, \ldots, p \qquad a = p+1, \ldots, 9$$

$$ds^2 = \omega \eta_{AB} dx^A dx^B + \omega^{-1} dx^a dx^a$$

$$G_s = \omega^{(p-3)/2} g_s \qquad C_{(p+1)} = \omega^2 g_s^{-1} dx^0 \wedge \cdots \wedge dx^p$$

$$S_{\mathrm{D}p} = -T_p G_s^{-1} \int d^{p+1}\sigma \sqrt{-\det(g_{\alpha\beta} + F_{\alpha\beta})} + T_p \int C_{(p+1)}$$

$$\xrightarrow{\omega\to\infty} T_p g_s^{-1} \int d^{p+1}\sigma \left(-\tfrac{1}{2}\partial_\alpha x^a \partial^\alpha x^a - \tfrac{1}{4} F_{\alpha\beta}F^{\alpha\beta}\right)$$

- Non-abelian extension: SYM
- T-duality relates to D0-brane limit. Also have e.g. S-duality of D1 limit

$\longrightarrow$ BPS F1 decoupling limit leading to Non-Relativistic String Theory [Gomis, Ooguri 2000] [Danielsson, Guijosa, Kruczenski 2000]

$$ds^2 = \omega^2 \eta_{AB} dx^A dx^B + dx^a dx^a \quad G_s = \omega g_s \quad B_{(2)} = \omega^2 dx^0 \wedge dx^1$$

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Curved spacetime? Replace:

$$dx^A \longrightarrow \tau^A{}_\mu(x) dx^\mu$$

$$dx^a \longrightarrow e^a{}_\mu(x) dx^\mu$$

$$S_{\mathrm{D}p} = -T_p G_s^{-1} \int d^{p+1}\sigma \sqrt{-\det(g_{\alpha\beta} + F_{\alpha\beta})} + T_p \int C_{(p+1)}$$

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Upgrade to curved space

And apply to whole of string theory

Dp limit in curved spacetime:

$A = 0, 1, \ldots, p$ $\quad a = p+1, \ldots, 9$

'longitudinal' 'transverse'

$$ds^2 = (\omega \eta_{AB} \tau^A{}_\mu \tau^B{}_\nu + \omega^{-1} e^a{}_\mu e^a{}_\nu) dx^\mu dx^\nu$$

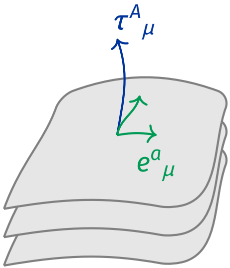
$$e^\Phi = \omega^{(p-3)/2} e^\varphi \qquad B_{(2)} = b_{(2)}$$

$$C_{(p+1)} = \omega^2 e^{-\varphi} \tau^0 \wedge \cdots \wedge \tau^p + c_{(p+1)}$$

$$C_{(q)} = c_{(q)} \qquad q \neq p+1$$

p-brane Newton-Cartan geometry

foliation of codimension $(p+1)$



Local symmetries: $SO(1,p)$ and $SO(9-p)$ rotations, Galilean p-brane boosts:

$\delta_G \tau^A = 0 \quad \delta_G e^a = \Lambda^a{}_A \tau^A$

$\delta_G c_{(p+1)} = \frac{1}{p!} e^{-\varphi} \Lambda^{aA} e^a \wedge \tau^{A_1} \wedge \cdots \wedge \tau^{A_p} \epsilon_{AA_1 \ldots A_p}$

and dilatations:

$\tau^A \to \Delta^{1/2} \tau^A \quad e^a \to \Delta^{-1/2} e^a$

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'longitudinal' $\quad$ 'transverse'

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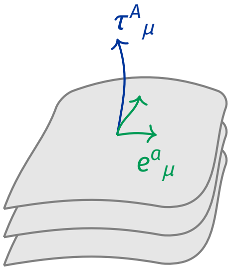
$$C_{(q)} = c_{(q)} \qquad q \neq p+1$$

- Limit of D-brane action: generalisation of SYM coupled to curved background defined by τ, e
e.g. $p = 0$ $S = \frac{1}{2\,\ell_s} \int d\tau\, e^{-\varphi} \frac{\dot{X}^\mu \dot{X}^\nu e^a{}_\mu e^a{}_\nu}{\dot{X}^\rho \tau_\rho{}^0} + \frac{1}{\ell_s} \int c_{(1)}$
- Limit of SUGRA action: divergences cancel or removed using auxiliary fields $\longrightarrow$ non-rel SUGRA

[M2 limit of 11-d SUGRA: Bergshoeff, CB, Lahnsteiner, Rosseel 2407.21648]

p-brane Newton-Cartan geometry

foliation of codimension $(p+1)$



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Lorentzian AdS/CFT from one decoupling limit

Asymptotic decoupling limit $\leftrightarrow$ near-horizon bulk limit

D-brane BPS decoupling limit $\longrightarrow$ SYM on Dp-branes
in open string picture — or matrix theory

Same limit in D-brane geometry $\longrightarrow$ near-horizon limit
closed string picture — AdS

$$X^A = \omega^{1/2} x^A \quad A = 0, \ldots, p$$
$$X^a = \omega^{-1/2} x^a \quad a = p+1, \ldots, 9$$
$$G_s = \omega^{(p-3)/2} g_s$$

$$ds^2 = H^{-1/2} \eta_{AB} dX^A dX^B + H^{1/2} dX^a dX^a$$

$$H = 1 + \frac{L^{7-p}}{R^{7-p}} \qquad L^{7-p} \sim \ell_s^{7-p} N G_s \qquad R^2 \equiv X^a X^a$$

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$$H = 1 + \frac{l^{7-p} \omega^{(p-3)/2}}{r^{7-p} \omega^{(p-7)/2}} = 1 + \omega^2 \left(\frac{l}{r}\right)^{7-p}$$

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$\omega \longrightarrow \infty$: near-horizon geometry

$p = 3$ $\mathrm{AdS}_5 \times S^5$ [Maldacena 1997]
$p \neq 3$ conformal to AdS_{p+2} ($p = 5$ Mink_7) [Itzhaki, Maldacena, Sonnenschein, Yankielowicz 1998]

$$ds^2 = \Omega(r) \eta_{AB} dx^A dx^B + \frac{1}{\Omega(r)} (dr^2 + r^2 d\Omega_{8-p}^2) \qquad \Omega(r) = (r/l)^{(7-p)/2}$$

$\Omega(r) \longrightarrow \infty$: geometry becomes non-Lorentzian asymptotically

Non-Lorentzian AdS/CFT from two decoupling limits

Take Dp limit of Dq solution, then near-horizon Dq limit (or vice versa)

# directions		m	n	
Dq geometry	$\times$	$\times$	–	–
Dp limit	$\times$	–	$\times$	–
index	α	I	u	i

near-horizon limit: $m = n = 0$

$$ds^2 = \omega (H^{-1/2}\eta_{\alpha\beta} dx^\alpha dx^\beta + H^{1/2} dx^u dx^u)$$
$$+\omega^{-1}(H^{-1/2} dx^I dx^I + H^{1/2} dx^i dx^i)$$
$$C_{(q+1)} = \omega^{\frac{4-m-n}{2}} g_s^{-1} H^{-1} dx^0 \wedge \cdots \wedge dx^q$$
$$e^\Phi = \omega^{\frac{p-3}{2}} g_s H^{\frac{3-q}{4}}$$

Limit needs: m+n=4 i.e. underlying brane intersection is 1/4 BPS

Also ensures $\omega^2 \tau \wedge \cdots \wedge \tau$ is closed (constant) $\longrightarrow$ can include in $C_{(p+1)}$

$$H = 1 + \frac{l^{7-q}\omega^{\frac{p+q-10}{2}}}{(x^u x^u + \omega^{-2} x^i x^i)^{\frac{7-q}{2}}}$$

$l^{7-q} \sim N g_s \ell_s^{7-q}$

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- Smear on x^u directions $\Longrightarrow H = 1 + \left(\frac{\tilde{l}}{||x^i||}\right)^{7-q-n}$

depends on transverse directions

- Rescale $N \longrightarrow \omega^{\frac{10-p-q}{2}} N \Longrightarrow H = 1 + \left(\frac{l}{||x^u||}\right)^{7-q}$

depends on longitudinal directions

Example: D1 limit of D3

Dual to quantum mechanics on BPS monopole moduli space

# directions	1	3	1	5
D3 geometry	×	×	–	–
D1 limit	×	–	×	–
index	0	I	4	i

$$\tau^A = (H^{-1/4}\mathrm{d}x^0, H^{1/4}\mathrm{d}x^4)$$

$$e^a = (H^{-1/4}\mathrm{d}x^I, H^{1/4}\mathrm{d}x^i)$$

$$c_{(4)} = g_s^{-1}H^{-1}\mathrm{d}x^0 \wedge \cdots \wedge \mathrm{d}x^3$$

$$e^\varphi = g_s$$

D3 near-horizon limit: $H = \left(\frac{l}{||x^i||}\right)^3$ (after smearing) or $H = \left(\frac{l}{x^4}\right)^4$ (after scaling N)

depends on transverse dirs — depends on longitudinal direction

Holographic dual? take a corresponding limit of $\mathcal{N} = 4$ SYM [Lambert, Smith 2024]

i.e. apply two BPS decoupling limits to D3-brane theory

$\longrightarrow$ localises on solutions of Bogomolny equation: QM on monopole moduli space

Bosonic Killing symmetries of smeared solution match symmetries of FT limit

Expect SUSY requires geometric constraints – need full D1 limit of type IIB SUGRA

Indirect support from brane solutions of M2 limit of 11-d SUGRA [CB 2503.21577]

Outlook

• Novel non-Lorentzian AdS/CFT: proposals accumulate, what to calculate?

[Lambert, Smith 2024 ×3] [CB, Lahnsteiner, Obers, Yan 2024 & 2025] [Harmark, Lahnsteiner, Obers 2025]
[Gomis, Gomis, Kamimura 2005] [Fontanella, García 2024]

Non-Lorentzian SUGRA – role of smeared vs scaled solutions?
Beyond SUGRA – light excitations in bulk not strings (in principle) but 'matrix *p*-brane theory' in curved backgrounds.

• Matrix theory revisited: understand via (e.g.) non-relativistic string theory? Curved backgrounds? (via holography?)

• Through the looking glass: timelike T-duality of D0 limit $\longrightarrow$ D(-1) decoupling limit and IKKT matrix model. Further T-dualities give Carroll ($c \longrightarrow 0$) limits associated with Euclidean D-branes. de Sitter holography? [Hull 1998] [CB, Obers, Yan 2506.xxxxx]

• New $T\bar{T}$-like deformations: non-abelian, quantum, applications?

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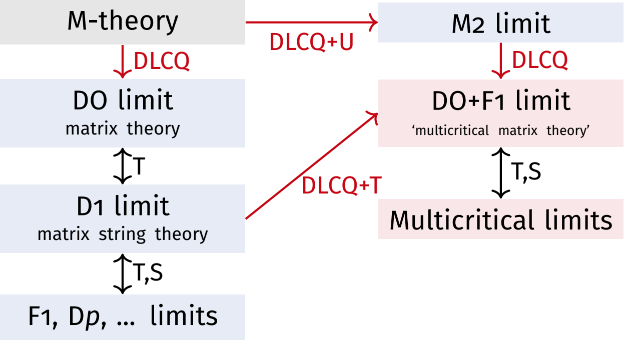
Thanks for listening!

extra slides

AdS/CFT as DLCQ^n/DLCQ^m

Conjecture: AdS/CFT can be organised as DLCQ^n/DLCQ^m, $m > n$

In terms of the non-Lorentzian nature of the bulk/asymptotic geometries



DLCQ^0: Lorentzian geometry

DLCQ^1: 1/2 BPS decoupling limits $\longrightarrow$ non-Lorentzian geometry

DLCQ^2: 1/4 BPS decoupling limits $\longrightarrow$ more non-Lorentzian geometry

[CB, Lahnsteiner, Obers, Yan 2311.10654]

[Gomis, Yan 2311.10565]

[de Boer, Dijkgraaf, Harmark, Obers unpublished]

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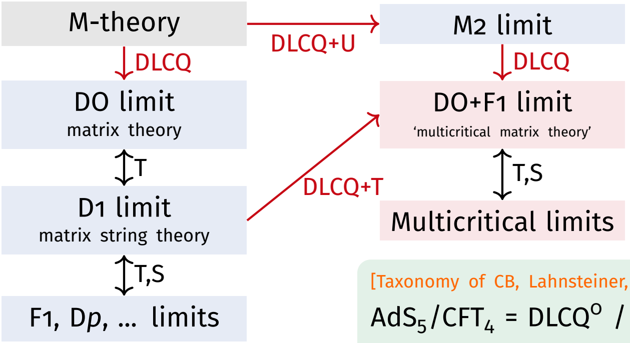

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[Taxonomy of CB, Lahnsteiner, Obers, Yan 2410.3591]

$\text{AdS}_5/\text{CFT}_4 = \text{DLCQ}^0$ / DLCQ^1

near-horizon bulk limit / asymptotic BPS decoupling limit

$\text{AdS}_3/\text{CFT}_2 = \text{DLCQ}^0$ / DLCQ^2

near-horizon bulk limit / asymptotic D1-D5 multicritical limit

New non-Lorentzian AdS/CFT: $\text{DLCQ}^1/\text{DLCQ}^2$

near-horizon and non-Lor bulk limit / asymptotic Dp-Dq multicritical limit

[CB, Lahnsteiner, Obers, Yan 2311.10654]
[Gomis, Yan 2311.10565]
[de Boer, Dijkgraaf, Harmark, Obers unpublished]

BPS decoupling limits in reverse are (generalised) $T\bar{T}$ deformations

$T\bar{T}$ and non-relativistic string theory

Deforming non-relativistic string theory to the relativistic theory [CB 2002.12413]

$$G_{\mu\nu} = \omega^2 \tau_{\mu\nu} + E_{\mu\nu} \qquad B_{(2)} = -\omega^2 \tau^0 \wedge \tau^1 + b_{(2)} \qquad e^{\Phi} = \omega e^{\varphi}$$

$$\tau_{\mu\nu} = \tau^A{}_\mu \tau^B{}_\nu \eta_{AB} \qquad E_{\mu\nu} = e^a{}_\mu e^a{}_\nu \qquad A = 0, 1 \qquad a = 2, \ldots, 9$$

$$S_{\rm NG} = -\int d^2\sigma \sqrt{-\det(\omega^2 \tau_{\alpha\beta} + E_{\alpha\beta})} - \int d^2\sigma (-\omega^2 \tau^0 \wedge \tau^1 + b_{(2)})$$

Deformation parameter $\lambda = \omega^{-2}$ really $\lambda = \omega^{-2}(2\pi\alpha')$

NRST limit: $S_{\rm NG}(\lambda = 0) = -\frac{1}{2}\int d^2\sigma \sqrt{-\det\tau}\,\tau^{\alpha\beta} E_{\alpha\beta} - \int d^2\sigma\, b_{(2)}$

Flat + static gauge:
$\tau_{\alpha\beta} = \eta_{\alpha\beta}$, $E_{\alpha\beta} = \partial_\alpha X^a \partial_\beta X^a$
$\longrightarrow$ free bosons

Deform to relativistic string theory: $\lambda \neq 0$

Classical flow equation for Lagrangian

$$\boxed{\frac{\partial L}{\partial \lambda} = \frac{1}{2}\sqrt{-\det\tau}\,\det(T_{\alpha\beta})}$$

energy-momentum tensor

$$T_{\alpha\beta} = -\frac{2}{\sqrt{-\det\tau}}\frac{\partial L}{\partial \tau^{\alpha\beta}}$$

$T\bar{T}$ deformation!

[Zamolodchikov 2004; Smirnov, Zamolodchikov 2016]
[Cavaglià et al 2016; Bonelli et al 2018]

$T\bar{T}$-like deformations from D-brane decoupling limits

Deforming matrix theory to DBI

$$G_{\mu\nu} = \omega\tau_{\mu\nu} + \omega^{-1}E_{\mu\nu} \qquad C_{(p+1)} = -\omega^2\tau^0\wedge\cdots\wedge\tau^p + c_{(p+1)} \qquad C_{(q)} = c_{(q)} \qquad B_{(2)} = b_{(2)}$$

$$\tau_{\mu\nu} = \tau^A{}_\mu\tau^B{}_\nu\eta_{AB} \qquad E_{\mu\nu} = e^a{}_\mu e^a{}_\nu \qquad A = 0,\dots,p \qquad a = p+1,\dots,9 \qquad e^\Phi = \omega^{(p-3)/2}e^\varphi$$

$$S_{\mathrm{DBI}} = \int d^{p+1}\sigma e^{-\varphi}\mathcal{L}(\lambda) + \int \Big(e^{\mathcal{F}}\sum_q c_{(q)}\Big)\Big|_{p+1} \qquad \mathcal{F} = F + b_{(2)}$$

$$\mathcal{L}(\lambda) = \frac{1}{\lambda}\left(\sqrt{-\det\tau} - \sqrt{-\det(\tau + \lambda^{1/2}\mathcal{F} + \lambda E)}\right) \qquad \lambda = \omega^{-2} \qquad \text{really } \lambda = \omega^{-2}/T_p$$

$$\mathcal{L}(\lambda = 0) = \sqrt{-\det\tau}\left(-\tfrac{1}{2}\tau^{\alpha\beta}E_{\alpha\beta} - \tfrac{1}{4}\tau^{\alpha\beta}\tau^{\gamma\delta}\mathcal{F}_{\alpha\gamma}\mathcal{F}_{\beta\delta}\right) \qquad \text{flat background: Maxwell + free scalars}$$

Computing $\frac{\partial\mathcal{L}}{\partial\lambda}$ and $T_{\alpha\beta}$ can derive new classical flow equations
These give $T\bar{T}$-like deformations (at least classically)

some related flow equations in [Ferko et al 2023] [Morone, Negro, Tateo 2024] [Tsolakidis 2024]

New $T\bar{T}$ flow equations from p-brane limits

Summarising with $\tau_{\alpha\beta} = \eta_{\alpha\beta}$, $\mathcal{T} = \eta^{-1}T$ [CB, Lahnsteiner, Obers, Yan 2410.3591]

- $\mathcal{F} = 0$. Flow equation such that free scalars $\longrightarrow$ Dirac-Nambu-Goto action

$$\frac{\partial \mathcal{L}}{\partial \lambda} = \frac{1}{2\lambda^2}\left[\mathrm{tr}\left(\mathbb{1} - \lambda\mathcal{T}\right) - \left(p-1\right)\left[\det(\mathbb{1} - \lambda\mathcal{T})\right]^{\frac{1}{p-1}} - 2\right]$$

2-d ($p = 1$): usual $T\bar{T}$ using $\mathrm{tr}\mathcal{T} = \lambda \det \mathcal{T}$

1-d ($p = 0$): matches 1-d $T\bar{T}$ deformation of [Gross, Kruthoff, Rolph, Shagoulian 2019] obtained by dimensional reduction. Deforming the point particle from non-relativistic to relativistic is $T\bar{T}$!

- $\mathcal{F} \neq 0$. Flow equations such that free scalars + Maxwell $\longrightarrow$ abelian DBI

$$p = 1: \quad \frac{\partial \mathcal{L}}{\partial \lambda} = -\frac{1}{2\lambda}\left(\mathrm{tr}\,\mathcal{T} + \sqrt{-\det\mathcal{F}}\sqrt{-\mathrm{tr}\,\mathcal{T} + \lambda\det\mathcal{T}}\right)$$

$$p = 2: \quad \frac{\partial \mathcal{L}}{\partial \lambda} = \frac{1}{4}\left[\mathrm{tr}\left(\mathcal{T}^2\right) - \left(\mathrm{tr}\,\mathcal{T}\right)^2\right] + \frac{\lambda}{2}\det\mathcal{T}$$

Geometric $T\bar{T}$ and AdS/CFT

The near-horizon geometry is a geometric deformation of non-Lorentzian geometry

D-brane near-horizon geometry:

$$ds^2 = \Omega(r)\eta_{AB}dx^A dx^B + \frac{1}{\Omega(r)}dx^a dx^a \qquad \Omega(r) = (r/l)^{(7-p)/2} \quad r^2 \equiv x^a x^a$$

$$C_{(p+1)} = \Omega(r)^2 g_s^{-1} dx^0 \wedge \ldots dx^p \qquad e^{\Phi} = \Omega(r)^{(p-3)/2} g_s$$

$\Omega(r) \longrightarrow \infty$: asymptotic geometry is flat non-Lorentzian $\tau^A = dx^A$ $e^a = dx^a$