MA22S3 Tutorial Sheet 5

1–3 November 2017

**Discrete Fourier Transform:**
A function $f(t)$ has been sampled at the regularly spaced points $t_k = 2\pi k/N$, for $k = 0, 1, \ldots, N - 1$, and the values $f(t_k) = f_k,$ have been measured.

It is possible to write a function interpolating among these points that takes the form

$$f(t) = \sum_{m=0}^{N-1} c_m e^{imt}.$$  

where the coefficients $c_m$ are given by

$$c_m = \frac{1}{N} \sum_{k=0}^{N-1} e^{-\frac{2\pi imk}{N}} f_k.$$

**Questions:**

1. Suppose we wish to perform a Discrete Fourier Transform for a function that has been sampled at 4 points, with the values

   $f(0) = 10$
   $f\left(\frac{\pi}{2}\right) = -5$
   $f(\pi) = 8$
   $f\left(\frac{3\pi}{2}\right) = -7$

   Compute the function given by the Discrete Fourier Transform, and identify the dominant frequency mode.

2. Evaluate the following integrals, and show your work.

   (a) $\int_0^1 \delta(x + 2) x^2 \, dx$
   (b) $\int_0^1 [\delta(x + 2) + \delta(x - 2)] x^2 \, dx$
   (c) $\int_{-\infty}^{\infty} \delta\left(\frac{x}{4}\right) (x + 3) \, dx$
   (d) $\int_{-1}^{1} \left[ \frac{d}{dx}\delta(x) \right] \sin x \, dx$
   (e) $\int_{-\infty}^{\infty} \delta(x - 1) \delta(x + 1) \, dx$
   (f) $\int_{-\infty}^{\infty} \delta(x^2 - 1) (x + 3) \, dx$