**Formulas:**

- The real Fourier series expansion of a function \( f(t) \) of fundamental period \( L \) can be written as

\[
f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \left( \frac{2\pi nt}{L} \right) + \sum_{n=1}^{\infty} b_n \sin \left( \frac{2\pi nt}{L} \right),
\]

where the coefficients are given by the Euler formulas:

\[
a_0 = \frac{2}{L} \int_{-L/2}^{L/2} f(t) \, dt
\]

\[
a_n = \frac{2}{L} \int_{-L/2}^{L/2} f(t) \cos \left( \frac{2\pi nt}{L} \right) \, dt
\]

\[
b_n = \frac{2}{L} \int_{-L/2}^{L/2} f(t) \sin \left( \frac{2\pi nt}{L} \right) \, dt
\]

- The complex Fourier series expansion of a function \( f(t) \) of period \( L \) can be written as

\[
f(t) = \sum_{n=-\infty}^{\infty} c_n e^{i \frac{2\pi nt}{L}},
\]

where the coefficients are given by

\[
c_n = \frac{1}{L} \int_{t_0}^{t_0+L} f(t) e^{-i \frac{2\pi nt}{L}} \, dt.
\]

- Parseval’s Theorem: For a function of period \( L \) whose real and complex Fourier series expansions are written in the forms above, the following equations are true:

\[
\frac{1}{L} \int_{t_0}^{t_0+L} f(t)^2 \, dt = \left( \frac{a_0}{2} \right)^2 + \frac{1}{2} \sum_{n=1}^{\infty} (a_n^2 + b_n^2)
\]

\[
\frac{1}{L} \int_{t_0}^{t_0+L} f(t)^2 \, dt = \sum_{n=-\infty}^{\infty} |c_n|^2
\]
Questions:

1. Sketch and write formulas for (a) the simple periodic extension, (b) the half-range even expansion, and (c) the half-range odd expansion of the following function.

\[ f(t) = e^{-t}, \quad \text{for } 0 < t < 1. \]

2. Compute the full complex Fourier series of the simple periodic extension in the previous problem, and write all the terms with \(|n| \leq 3\) explicitly.

3. In the previous tutorial, we computed the Fourier series of the following function:

\[
\begin{align*}
  f(t) & = t \quad \text{for } -\pi < t \leq \pi, \\
  f(t) & = f(t + 2\pi).
\end{align*}
\]

The following Fourier coefficients were obtained:

\[ a_0 = 0, \quad a_{n \geq 1} = 0, \quad b_n = -\frac{2}{n} (-1)^n. \]

Use Parseval’s Theorem to evaluate the sum of the infinite series

\[ \sum_{n=1}^{\infty} \frac{1}{n^2}. \]