MATHEMATICAL NOTES

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ANOTHER PROOF OF CAUCHY'S GROUP THEOREM

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Since $ab=1$ implies $ba=b(ab)b^{-1}=1$, the identities are symmetrically placed in the group table of a finite group. Each row of a group table contains exactly one identity and thus if the group has even order, there are an even number of identities on the main diagonal. Therefore, $x^2=1$ has an even number of solutions.

Generalizing this observation, we obtain a simple proof of Cauchy's theorem. For another proof see [1].

CAUCHY'S THEOREM. *If the prime p divides the order of a finite group G, then G has kp solutions to the equation $x^p=1$.*

Let G have order n and denote the identity of G by 1. The set

$$S = \{(a_1, \cdots, a_p) \mid a_i \in G,\ a_1a_2 \cdots a_p = 1\}$$

has n^{p-1} members. Define an equivalence relation on S by saying two p-tuples are equivalent if one is a cyclic permutation of the other.

If all components of a p-tuple are equal then its equivalence class contains only one member. Otherwise, if two components of a p-tuple are distinct, there are p members in the equivalence class.

Let r denote the number of solutions to the equation $x^p=1$. Then r equals the number of equivalence classes with only one member. Let s denote the number of equivalence classes with p members. Then $r+sp=n^{p-1}$ and thus $p|r$.

Reference

1. G. A. Miller, On an extension of Sylow's theorem, Bull. Amer. Math. Soc., vol. 4, 1898, pp. 323–327.

A REMARK ON BOUNDED FUNCTIONS

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Denote by E the class of functions regular and bounded by unity in $|z|<1$. Denote by E^* the subclass of functions of E which are in addition univalent in $|z|<1$. Analogies of various inequalities which are known to hold for functions in the class E have been obtained for functions of the class E^*. For example, it is known [3] that there exist functions in E for which the sequence $\{a_0+\cdots+a_n\}$ $(f(z)=\sum a_nz^n)$ is unbounded. On the other hand, it is shown by Fejér in [1] that if $f\in E^*$ then $|a_0+\cdots+a_n|<1+(1/\sqrt{2})$ for all n.