Problem Solvers Week 1

1. A monk walks up a mountain, taking 12 hours to complete the trip. He then sleeps for 12 hours and then descends from the top taking 12 hours more. Prove at some point on both days he was at exactly the same height at exactly the same time.

2. Prove that
$$\frac{1}{\log_2 N} + \frac{1}{\log_3 N} + \frac{1}{\log_4 N} + \cdots + \frac{1}{\log_{100} N} = \frac{1}{\log_{100!} N}$$

3. Which of the expressions,
$$(1 + x^2 - x^3)^{1000} \text{ or } (1 - x^2 + x^3)^{1000}$$
will have the larger coefficient for x^{20} after expansion and collecting of terms?

4. $\int_0^\infty \frac{\ln x}{1+x^2}\, dx$

5. The integer A consists of 666 threes, and the integer B has 666 sixes. What digits appear in the product $A \times B$?

6. Prove the following interesting property of the natural numbers; For all numbers N there exist integers x and y such that $x^y + y^x = N$ as in the case where N is 17 we have $2^3 + 3^2 = 17$.

7. Which integers have the following property? If the final digit is deleted, the integer is divisible by the new number.

8. Bonus Problem (Daron Anderson)
Identify the following (finite) integer sequence:

$$7, 9, 10, 12, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32,$$

$$33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 47, 48, 49, 50, 52, 54, 55, 59, 64$$

A clue would be to think about the Japanese cartoons you might know.