Mathematical Equations of Physics

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1 Introduction

This is an attempt to create a list of all possible “give definition” or “prove theorem” type questions that can be asked based on the notes for the course so far. It includes all exercise/problem type questions given so far in lectures, but doesn’t include any homework exercises (yet). The emphasis is on providing a complete overview of what has been covered in the course so far, in chronological order, as well as an easy way to test how well the material is understood, and so I make no attempt to mimic realistic ‘exam’ questions, nor do I attempt to divide the questions into easy and hard. For the type of questions you’ll actually see on an exam, see one of the several other lists of questions available.

I intend to make one of these for at least each of the Maths courses, and possibly for the Physics classes too.

1.1 Week 1

Todo

1. Give the formula for the Fourier Series.
2. Why is the Fourier series better for periodic functions than the Taylor Series?
3. Explain how cos and sin are the basis vectors, in some sense, of the Fourier Series.
4. Derive the formulae for a_0, a_n and b_n
5. Find the Fourier Series for the function

$$f(x) = \begin{cases} 1 & \text{if } 0 < x \leq \pi \\ -1 & \text{if } -\pi < x \leq 0 \end{cases}$$

6. What is the Gibbs phenomenen?
7. What is Dirichlet's Theorem proper?
8. What is the common extension of Dirichlet's Theorem? (which is often just called Dirichlet's Theorem)?
9. Give the expressions for complex sin and cos.
10. What is the complex dot product?
11. Give the formula for the complex Fourier Series.
12. Derive the forumla for c_n.
13. What conditions on c_n must be met if we wish the complex Fourier series to be real valued?

1.2 Week 2

Todo

1. Using the real Fourier series for the function

$$f(x) = \begin{cases} 1 & \text{if } 0 < x \leq \pi \\ -1 & \text{if } -\pi < x \leq 0 \end{cases}$$

find the complex Fourier coefficients.

2. Derive the complex Fourier coefficients for the formula above using the formula for c_n.
3. Find the complex Fourier Series for the function

$$f(t) = e^t, -\pi < t < \pi$$

4. Is the function above real valued? (told from the coefficients)
5. State and prove Parseval's Theorem.
6. Show that the space of all periodic functions is a vector space. (prove axioms)
7. Define an inner product space.
8. What is the inner product we use? Show it satisfies the axioms.
9. Give the formula for the Fourier transform.
10. Give the function for the Fourier transform coefficients.

Week 3

Todo

1. How does the Fourier transform functions of time and (say) position?
2. What condition must be met for $\tilde{f}(k)$ to be well defined?
3. Define a Schwartz function
4. Integrate the following function:
$$\int_{-\infty}^{+\infty} e^{-\frac{1}{2}at^2} dt$$
5. Find the Fourier transform for the function
$$f(x) = \begin{cases} 1 & -1 < t < 1 \\ 0 & \text{otherwise} \end{cases}$$
6. Suppose we have a function with a certain width, what will the Fourier transform look like?
7. Which of the following functions are or aren't Schwartz functions and why?
$$f(t) = e^{-t} f(t) = e^{-|t|}$$
8. Find $\tilde{f}(k)$ for the Schwartz function above.
9. Suppose we have $f(t)$ and $f'(t)$, how do their respective Fourier transforms relate?
10. What can we say about differentiation in general with regard to Fourier transforms?
11. Find the solution to the equation $\dfrac{\partial^2 f}{\partial t^2} + af = g$ in Fourier Space.
12. Suppose we have ft and $f(at)$, how do their respective Fourier transforms relate?
13. State *Plancheral's Theorem.*
14. Find a value for $\displaystyle\int_{-\infty}^{+\infty} e^{-2|t|}$ TODO FINISH THIS (WHAT@S GOING ON?)

Week 4

1. Why do we require Schwartz functions for the Fourier Transform? Show why they meet the requirements.

2. Show that the function $\tilde{f}(k) = \dfrac{1}{2\pi}$ is not a Schwartz function.

3. Ignoring that it isn't a Schwartz function, find $\delta(t) = \displaystyle\int\limits_{-\infty}^{+\infty} \frac{1}{2\pi} e^{ikt} dk$