Mechanics Questions

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1 Introduction

This is an attempt to create a list of all possible "give definition" or "prove theorem" type questions that can be asked based on the notes for the course so far. It includes all exercise/problem type questions given so far in lectures, but doesn't include any homework exercises (yet). The emphasis is on providing a complete overview of what has been covered in the course so far, in chronological order, as well as an easy way to test how well the material is understood, and so I make no attempt to mimic realistic 'exam' questions, nor do I attempt to divide the questions into easy and hard. For the type of questions you'll actually see on an exam, see one of the several other lists of questions available.

I intend to make one of these for at least each of the Maths courses, and possibly for the Physics classes too.

1.1 Week 1

This week introduced foundational concepts like generalised coordinates, Einstein notation and the degrees of freedom of a system. We were also introduced to the principle of least action, and the week culminated in a derivation of the Euler-Lagrange equations.

1. Explain how the coordinates of a point are given in Einstein notation.

2. Find the metric of $\mathbb{R}^3$ in cartesian, cylindrical and polar coordinates.

3. Define the degrees of freedom of a system.

4. How is the number of degrees of freedom of a system determined?

5. What is the Lagrangian of a system?

6. What is the action of a system?
7. State Hamilton's principle of least action.
8. Derive (from first principles) the equations of motion for the Lagrangian

$$L = \frac{m}{2}\dot{\bar{r}}^2$$

9. Derive (from first principles) the equations of motion for the Lagrangian

$$L = \frac{m}{2}\ddot{\bar{r}}^2 + \bar{F}(t)\dot{\bar{r}}$$

10. Suppose we have some function $U = U(\bar{r}_1, \bar{r}_2, \ldots, \bar{r}_n)$. What is δU? (i.e. Show you can do the partial derivative properly.)
11. Derive the Euler-Lagrange equations.

1.2 Week 2

We looked at some stuff relating to the interactions of systems. We also looked at the general method for finding the Lagrangian of a system with constrained motion. The week culminated in the statement of Noether's Theorem. I'll probably completely rewrite this section once I actually understand what's going on.

1. Suppose we have two systems A,B. How do we add together the Lagrangians for the two systems?
2. What can we say about the multiplicative properties of the Lagrangian?
3. What can we say about the Lagrangian $\tilde{L}(q, \dot{q}, t) = L(q, \dot{q}, t) + \frac{d}{dt}f(q, t)$ and why?
4. Show how this allows us to add any constant to potential energy without altering the EOMs.
5. For the Lagrangian

$$L = \sum_{a=1}^{N} \frac{m_a}{2}\bar{v}_a^2 - U(\bar{r}_1, \ldots, \bar{r}_n)$$

   (a) What are some of the implications of the fact that U has no explicit time dependence?

   (b) Derive the EOM.
   (c) Rewrite the KE term in terms of arbitrary generalised coordinates.

6. What sort of form would the Lagrangian for two interacting systems A and B have?
7. What can we say about the motion of B if the reaction of B on the action of A is negligible?
8. What is the method for finding the Lagrangian of a system with constrained motion?
9. Find the Lagrangian for a coplanar double pendulum in a uniform gravitational field.
10. What is an Integral of Motion?
11. How many Integrals of Motion are there?
12. What is an additive Integral of Motion and why is it useful?
13. In rough terms, what does Noether's Theorem state?

1.3 Week 3

This week we continued looking at the symmetries and associated conservation laws of systems, as well as proving Noether's theorem.

1. Something something show linear momentum / conservation *FIX THIS*
2. Prove Noether's Theorem

1.4 Week 4

TODO

1.5 Week 5

TODO

1.6 Week 6

1.7 Reading Week

TODO