we have

$$\Delta W = a\int_{r_{min}}^{\infty} dr\ \frac{1}{r^4\sqrt{\frac{1}{r_{min}}-\frac{1}{r}}} = a\int_{r_{min}}^{\infty} dr\ \frac{1}{r^{7/2}\sqrt{\frac{r}{r_{min}}-\frac{1}{r}}}$$

We make the substitution

$$y = \frac{\sqrt{r}}{\sqrt{r_{min}}} \Rightarrow r = y^2 r_{min}$$

$$\frac{dy}{dr} = \frac{1}{2}\frac{1}{\sqrt{r\, r_{min}}} \Rightarrow dy = dr\,\frac{1}{2\sqrt{r_{min} r}}, \quad dr = 2\sqrt{r_{min}}\, r^{1/2}\, dy$$

limits: $y(\infty) = \infty$, $y(r_{min}) = 1$

so our integral becomes

$$\Delta W = a\int_1^{\infty} 2(r_{min})^{1/2} r^{1/2}\, dy\ \frac{1}{r^{7/2}\sqrt{\frac{r}{r_{min}}-1}}$$

$$= a\int_1^{\infty} 2(r_{min})^{1/2}\ \frac{1}{r^{6/2}\sqrt{\frac{r}{r_{min}}-1}}\ dy$$

$$= a\int_1^{\infty} 2(r_{min})^{1/2}\ \frac{1}{y^6\sqrt{y^2-1}\cdot r_{min}^{6/2}}\ dy$$

$$= a\,\frac{2}{r_{min}^{5/2}}\int_1^{\infty} \frac{1}{y^6\sqrt{y^2-1}}\ dy$$

as required.