# MA3441 Problem Set 2

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## Disclaimer

Everything I've written might be completely wrong. Check that what I wrote matches with what Prof. Fry wrote, especially if you keep getting the wrong answer.

## Problems

1. The observable Q has the complete set of discrete normalizable eigenstates $|n\rangle$ with eigenvalues $q_n$:

$$Q|n\rangle = q_n|n\rangle, \langle n|m\rangle = \delta_{mn}$$

The completeness relation for this basis is

$$\sum_k |n\rangle\langle n| = 1 \ (\equiv \text{unit operator}).$$

Provide a derivation for each of the following inequalities:

(a) For any observable $P$, the expectation value of $P^2$ in the state $|n\rangle$, $\langle n|P^2|n\rangle$, satisfies

$$\langle n|P^2|n\rangle \geq |\langle n|P|m\rangle|^2$$

where $m$ is any state from the above basis.

(b) For any normalisable state vector $|\psi\rangle$,

$$\langle\psi|Q|\psi\rangle \geq q_0\langle\psi|\psi\rangle$$

where $q_0$ is the smallest of the eigenvalues $q_n$ belonging to the eigenstates $|n\rangle$ of $Q$.

(c) For any two normalised state vectors $|\psi\rangle$ and $|\varphi\rangle$, the real pat of their inner product, $\langle\varphi|\psi\rangle$ satisfies

$$-1 \leq \Re\langle\varphi|\psi\rangle \leq 1$$

(d) For any observable $P$ and normalised state vector $|\psi\rangle$,

$$\langle\psi|P^2|\psi\rangle \geq \langle\psi|P|\psi\rangle^2$$

2. The Hamiltonian operator for a two-state system is given by

$$H = a(|1\rangle\langle 1| - |2\rangle\langle 2| + |1\rangle\langle 2| + |2\rangle\langle 1|)$$

where $a$ is a real number with the dimension of energy. Find the energy eigenvalues and the corresponding energy eigenstates.

Ans:

$$|\sqrt{2}a\rangle = \frac{1}{2}(2+\sqrt{2})^{1/2}|1\rangle + \frac{1}{2}(2+\sqrt{2})^{1/2}(\sqrt{2}-1)|2\rangle$$

$$|-\sqrt{2}a\rangle = \frac{1}{2}(2+\sqrt{2})^{1/2}(1-\sqrt{2})|1\rangle + \frac{1}{2}(2+\sqrt{2})^{1/2}|2\rangle$$

3. If $A$,$B$ = Hermitian, show that $\langle\varphi|AB|\psi\rangle^* = \langle\psi|BA|\varphi\rangle$

4. Given the non-Hermitian matrix $\left(\begin{smallmatrix} 0 & 9 \\ 1 & 0 \end{smallmatrix}\right)$, Investigate

   (a) its eigenvalues;

   (b) its eigenvectors, i.e., find them;

   (c) their orthogonality;

   (d) and their completeness.