

Problem set 1

(3)

Pr. 5 (continued)

Independent coordinates:

- 1)  $\varphi$  is the angle between  $\vec{OA}$  and the y-axis
- 2)  $\theta$  is the angle between  $\vec{AB}$  and the z-axis
- 3)  $\psi$  is the angle between the x-axis and the projection of  $\vec{AB}$  onto the xy-plane

$\theta$  and  $\psi$  are the angles of the spherical coordinates

Solving the constraints

$$x_1 = 0; \quad y_1 = R \cos \varphi; \quad z_1 = R \sin \varphi \quad (\varphi \text{ can be negative})$$

$$x_2 = x_1 + l \sin \theta \cos \psi = l \sin \theta \cos \psi$$

$$y_2 = y_1 + l \sin \theta \sin \psi = R \cos \varphi + l \sin \theta \sin \psi$$

$$z_2 = z_1 + l \cos \theta = R \sin \varphi + l \cos \theta$$

Substituting into

$$L = \frac{1}{2} m_1 \dot{\vec{r}}_1^2 + \frac{1}{2} m_2 \dot{\vec{r}}_2^2 - m_1 g z_1 - m_2 g z_2$$

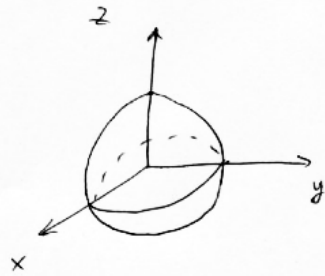
we get

$$L = \frac{R^2}{2} (m_1 + m_2) \dot{\varphi}^2 - m_2 l R \dot{\varphi} \left( (\cos \varphi \sin \theta + \cos \theta \sin \varphi \sin \psi) \dot{\theta} + \cos \varphi \sin \varphi \sin \theta \dot{\psi} \right) + \frac{1}{2} m_2 l^2 \left( \dot{\theta}^2 + \sin^2 \theta \dot{\psi}^2 \right) - g(m_1 + m_2) R \sin \varphi - g m_2 l \cos \theta$$

Problem set 1

(2)

Pr. 4



$$x^2 + y^2 + z^2 = R^2$$

$$L = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} m \dot{y}^2 + \frac{1}{2} m \dot{z}^2 - mgz$$

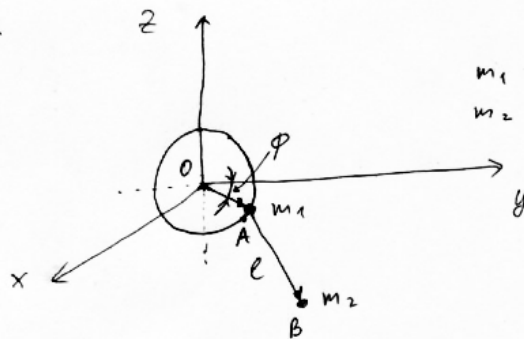
Constraint:  $x^2 + y^2 + z^2 = R^2$

Spherical coordinates:  $z = R \cos \theta$ ;  $x = R \sin \theta \cos \varphi$ ;  $y = R \sin \theta \sin \varphi$

$$L = \frac{1}{2} m (\dot{R}^2 + R^2 \dot{\theta}^2 + R^2 \sin^2 \theta \dot{\varphi}^2) - mgR \cos \theta = | \dot{R} = 0 | \Rightarrow$$

$$L = \frac{1}{2} m R^2 (\dot{\theta}^2 + \sin^2 \theta \dot{\varphi}^2) - mgR \cos \theta$$

Pr. 5



$x \perp$  the circle

$m_1$  has coordinates  $(x_1, y_1, z_1)$

$m_2$  has coordinates  $(x_2, y_2, z_2)$

Constraints:

$$C_1 = x_1 = 0 \quad (\text{circle lies in the } yz\text{-plane})$$

$$C_2 = y_1^2 + z_1^2 - R^2 = 0 \quad (m_1 \text{ is on the circle})$$

$$C_3 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2 - l^2 = 0 \quad (\text{distance between } m_1 \text{ and } m_2 \text{ is } l)$$

Problem set 1

(1)

Pr. 1.  $L = \frac{1}{2} m v_i^2 - \frac{\alpha}{r} = \frac{1}{2} m \dot{x}_i^2 - \frac{\alpha}{r}$

$$\delta L = -m \ddot{x}_i \delta x_i + \frac{\alpha}{r^2} \delta r = -m \ddot{x}_i \delta x_i + \frac{\alpha}{r^3} x_i \delta x_i = 0$$

$$\boxed{m \ddot{x}_i = \alpha \frac{x_i}{r^3}}$$

$$x_i \rightarrow O_{ij} x_j \Rightarrow v_i \rightarrow O_{ij} v_j$$

$$L \rightarrow L \text{ because}$$

$$x_i^2 \rightarrow O_{ij} x_j O_{ik} x_k = O_{ij} O_{ik} x_j x_k = \delta_{jk} x_j x_k = x_i^2$$

Pr. 2.  $L = \frac{1}{2} m_{ij} v_i v_j - \frac{1}{2} k_{ij} x_i x_j$

$$\delta L = -\frac{1}{2} (m_{ij} + m_{ji}) \ddot{x}_j \delta x_i - \frac{1}{2} (k_{ij} + k_{ji}) x_j \delta x_i = 0$$

$$\boxed{\frac{1}{2} (m_{ij} + m_{ji}) \ddot{x}_j + \frac{1}{2} (k_{ij} + k_{ji}) x_j = 0}$$

$$D=1 ; m \ddot{x} + kx = 0 \Rightarrow x = a \cos(\sqrt{\frac{k}{m}} t + \varphi)$$

Pr. 3.  $L = -m_0 c^2 \sqrt{1 - \frac{1}{c^2} \dot{x}_i^2}$

$$\delta L = m_0 c^2 \frac{\frac{1}{c^2} \dot{x}_i \delta \dot{x}_i}{\sqrt{1 - \frac{1}{c^2} \dot{x}_j^2}} = -m_0 \left( \frac{\dot{x}_i}{\sqrt{1 - \frac{1}{c^2} \dot{x}_j^2}} \right) \delta x_i = 0$$

$$\boxed{\left( \frac{\dot{x}_i}{\sqrt{1 - \frac{1}{c^2} \dot{x}_j^2}} \right) \dot{\phantom{x}} = 0} \Rightarrow \frac{v_i}{\sqrt{1 - \frac{1}{c^2} v^2}} = \text{const} \Rightarrow$$

$$\Rightarrow v_i = \text{const} ; v^2 < c^2$$