Problem set 1 ③

Pr. 5 (continued)

Independent coordinates:

1) ϕ is the angle between $\vec{OA}$ and the y-axis
2) θ is the angle between $\vec{AB}$ and the z-axis
3) φ is the angle between the x-axis and the projection of $\vec{AB}$ onto the xy-plane

θ and φ are the angles of the spherical coordinates

Solving the constraints

$x_1 = 0$; $y_1 = R\cos\phi$; $z_1 = R\sin\phi$ (ϕ can be negative)

$$x_2 = x_1 + \ell\sin\theta\cos\varphi = \ell\sin\theta\cos\varphi$$

$$y_2 = y_1 + \ell\sin\theta\sin\varphi = R\cos\phi + \ell\sin\theta\sin\varphi$$

$$z_2 = z_1 + \ell\cos\theta = R\sin\phi + \ell\cos\theta$$

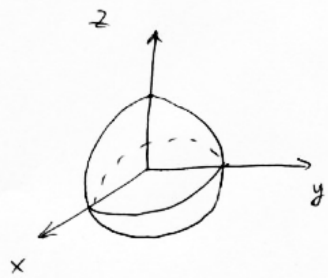
Substituting into

$$L = \frac{1}{2}m_1\dot{\vec{r}}_1^{\,2} + \frac{1}{2}m_2\dot{\vec{r}}_2^{\,2} - m_1 g z_1 - m_2 g z_2$$

we get

$$\boxed{\begin{aligned} L = {} & \frac{R^2}{2}(m_1+m_2)\dot{\phi}^2 - m_2\ell R\dot{\phi}\Big(\big(\cos\phi\sin\theta + \cos\theta\sin\phi\sin\varphi\big)\dot{\theta} + \\ & + \cos\varphi\sin\phi\sin\theta\,\dot{\varphi}\Big) + \frac{1}{2}m_2\ell^2\left(\dot{\theta}^2 + \sin^2\theta\,\dot{\varphi}^2\right) - \\ & - g(m_1+m_2)R\sin\phi - g m_2\ell\cos\theta \end{aligned}}$$

Pr. 4



$$x^2+y^2+z^2=R^2$$

$$L = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}m\dot{y}^2 + \frac{1}{2}m\dot{z}^2 - mgz$$

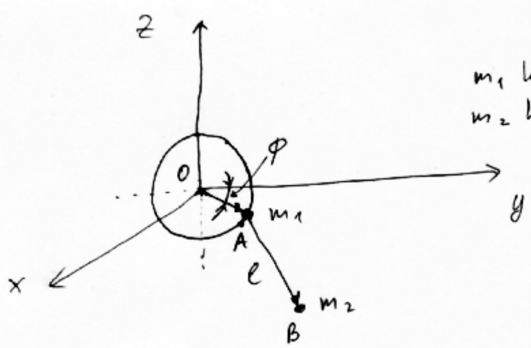
Constraint: $x^2+y^2+z^2=R^2$

spherical coordinates: $z = R\cos\theta$; $x = R\sin\theta\cos\varphi$; $y = R\sin\theta\sin\varphi$

$$L = \frac{1}{2}m\left(\dot{R}^2 + R^2\dot{\theta}^2 + R^2\sin^2\theta\,\dot{\varphi}^2\right) - mgR\cos\theta = \left|\dot{R}=0\right| \Rightarrow$$

$$\boxed{L = \frac{1}{2}mR^2\left(\dot{\theta}^2 + \sin^2\theta\,\dot{\varphi}^2\right) - mgR\cos\theta}$$

Pr. 5



$x \perp$ the circle

m_1 has coordinates (x_1, y_1, z_2)

m_2 has coordinates (x_2, y_2, z_2)

Constraints:

$C_1 = x_1 = 0$ (circle lies in the yz-plane)

$C_2 = y_1^2 + z_1^2 - R^2 = 0$ (m_1 is on the circle)

$C_3 = (x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2 - \ell^2 = 0$ (distance between m_1 and m_2 is ℓ).