Polarised Light

Andrew Mark Allen - 05370299

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Abstract

Various properties of polarised light were investigated. Malus' Law was verified experimentally in this experiment, and the reflectance of light off a prism at various angles of incidence was measured for light polarised both perpendicular and parallel to the plane of reflection. These results were compared to the theoretical predictions given by the Fresnel equations. Brewster's angle for glass was measured to be $59 \pm 1°$ and from this the refractive index of glass was found to be 1.66 ± 0.05.

Aims

- To verify Malus' Law experimentally
- To measure the intensity of light reflected from a prism at various angles of incidence for light polarised parallel, then perpendicular, to the plane of reflection.
- To determine Brewster's angle and the refractive index of the glass prism.

Introduction and Theory

Polarisation

Light waves are a form of electromagnetic wave energy consisting of electric and magnetic sinusoidal vibrations perpendicular to one another and to the direction of motion. A beam of light ordinarily consists of an immense number of such waves, and for any direction of motion, there are many possible planes intersecting that axis in which it is possible for the electric component of a light wave to be vibrating perpendicular to it.

A polariser, e.g. Polaroid, is a device which produces light in which the vibrations are constrained to occur in one direction alone. Such light is then called plane-polarised light.

Malus' Law

When linearly polarised light is passed through a polariser, the resulting light is polarised in the direction of the polariser. The intensity of the resulting light is dependent on the relative angle between the incident light and the plane of polarisation of the polariser. Only the component of the electric field which is along the polaroid transmission axis is transmitted, therefore we have

$$E_t = E_0 \cos\phi$$

Since $I_t = |E_t|^2$, we have

$$I_t = I_0 \cos^2\phi$$

which is the statement of Malus' Law.

Reflectance

Light is externally reflected when light crosses from a low refractive index medium to a high refractive index medium. The relative intensity of the reflected wave compared to the incident wave is called the *reflectance* and is described by the *Fresnel formulas.*

How light is reflected depends on the direction of polarisation of light. The *plane of incidence* is determined by an incoming ray and is perpendicular to the interface, and light can be thought of as a superposition of two linearly polarised components, one oscillating in the plane of incidence (*p-polarised*) and one oscillating perpendicular to it (*s-polarised*).

Brewster's Angle

At a particular angle of incidence for p-polarised light, the reflectance is exactly zero, no light is reflected. Hence, an unpolarised light reflected at this angle becomes s-polarised. This angle is called *Brewster's angle.*

The condition for Brewster's Angle is that the reflected ray is at a right angle to the transmitted ray, i.e. $\theta_r + \theta_t = 90$, however, $\theta_r = \theta_i$ by the law of reflection. Snell's law gives us

$$n_1 \sin\theta_i = n_2 \sin\theta_t$$

thus

$$n_1 \sin\theta_i = n_2 \sin(90 - theta_r) = n_2 \cos\theta_r$$

Thus $n1/n2 = tan\theta_i$, which allows us to calculate the refractive index of n_2 if we know n_1 and Brewster's angle.

Fresnel formulas for s- and p-component reflectance

$$R_S = \left(\frac{n_1 \cos\alpha - n_2 \cos\beta}{n_1 \cos\alpha + n_2 \cos\beta}\right)^2$$

$$R_P = \left(\frac{n_1 \cos\beta - n_2 \cos\alpha}{n_1 \cos\beta + n_2 \cos\alpha}\right)^2$$

Experimental Method

The experimental setup was as in Figure 1.

A Neon-Helium laser was used as a source of monochromatic light. A silicon photodiode mounted on a swivel arm was used to measure the intensity of reflected light, and a multimeter measured the voltage across the diode.

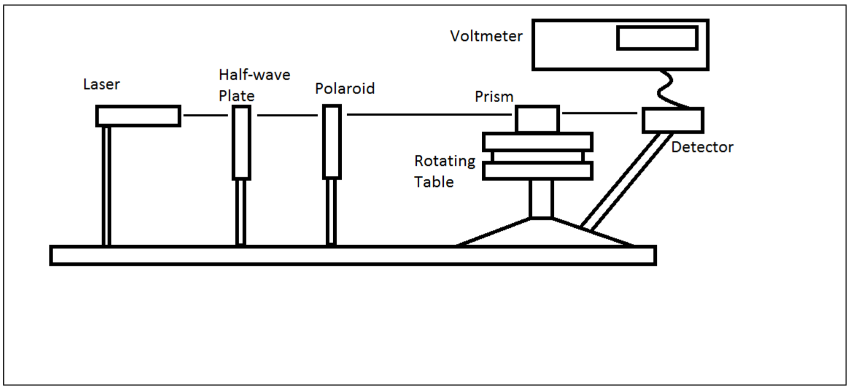

Figure 1: Experimental Setup

Malus' Law

For this, the prism and the half wave plate were removed from the apparatus. The photodiode was arranged so that the beam shone directly on it. The polaroid was rotated until a maximum was recorded on the multimeter, and this was taken to be a relative angle of 0°. The intensity reading for this angle was recorded, and the polariser was rotated 10°and the intensity reading recorded successively up to 180°. A graph was plotted for the intensity against the angle of incidence, and for $\cos^2$ of the intensity against the angle of incidence.

Reflectance

First, it was ensured that the plane of polarisation was perpendicular to the plane of incidence. An intensity reading was first taken without the prism (I_0). The prism was then replaced and intensity readings of the reflected light were then taken successively every 10°as the prism was rotated until readings were no longer possible.

Then, the half wave plate was placed between the laser and the prism in order to rotate the plane of rotation of the light by 90°. Thus, the light would now be polarised parallel to the plane of reflectance (since the laser was originally polarised vertically). Again, I_0 was measured without the prism in place, and successive readings were taken. Brewster's angle was also measured, by finding where the minimum intensity occured.

Graphs were plotted for the reflectance (I_r/I_0) as a function of the angle of incidence for both the parallel and perpendicular polarisations.

Results and Analysis

Malus' Law

According to Malus' Law, the intensity of the transmitted light should vary with $\cos^2(\theta)$ of the angle between the polariser and plane of polarisation of the light source.

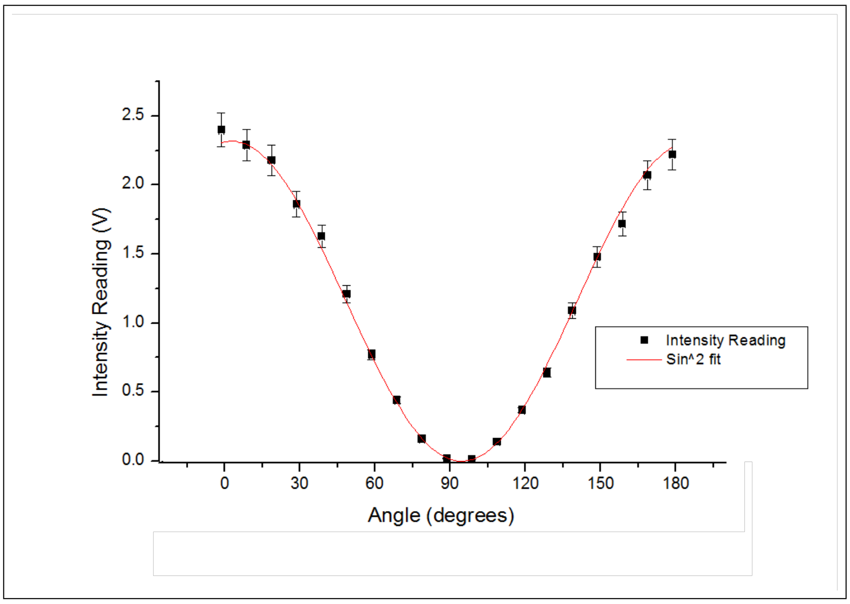

Figure 2: Intensity of transmitted light as a function of angle between polarised and plane of polariser of light.

The experimental results for the Malus' Law part of the experiment are shown in Figure 2. It can be seen how the intensity of light reduces to zero as the angle between the is approximately 90°, while the intensity is at a maximum at approximately 0 and 180°.

To see the Malus' Law relationship clearly, we took $\cos^2$ of the intensity. The results are in Figure 3 and as can be seen the result is not a straight line graph, but rather that of an ellipse symmetrical about the line $y = x$.

Recognising that this figure is consistent with two sinusoidally varying functions with different phase offsets, it was clear that the value we recorded as 90° (i.e. where the intensity was at a minimum) was incorrect. We were able to quantify exactly how off our results were by using non-linear regression to fit a $\cos^2$ function to the data and use the minimum from that to find out the offset required.

The fitted curve is visible on Figure 2. When the offset was added to the original function, the cos^2 of the intensity was plotted against the angle to produce Figure 4 where the linear relationship is clearly visible (R-squared for the fit was 0.995).

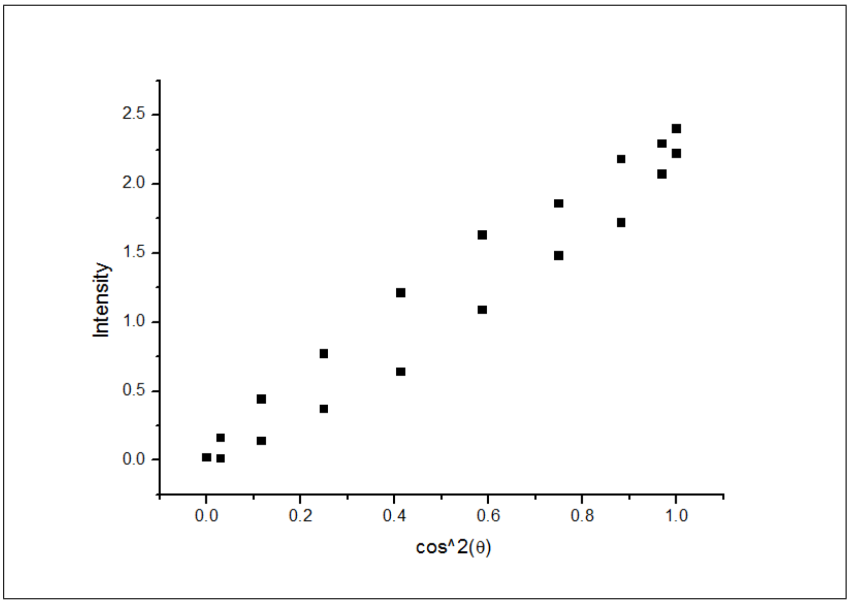

Figure 3: $\cos^2$ of intensity of transmitted light as a function of angle between polariser and plane of polarisation of light (before angle corrections).

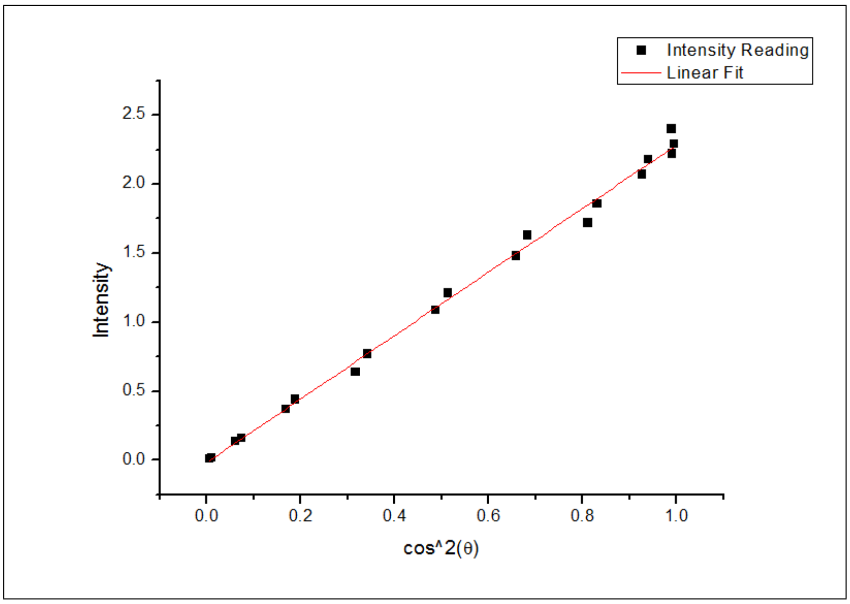

Figure 4: $\cos^2$ of intensity of transmitted light as a function of angle between polariser and plane of polarisation of light (after angle corrections).

Reflectance and Brewster's Angle

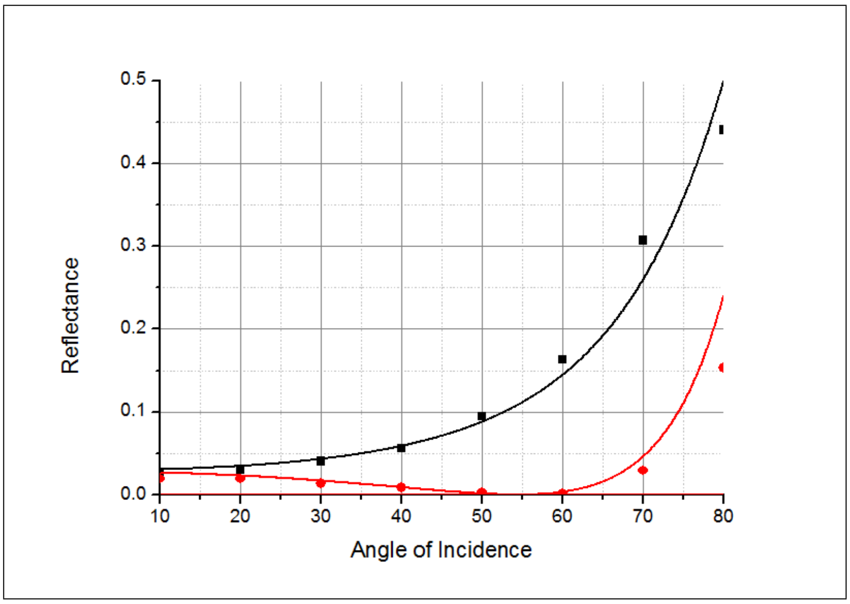

Figure 5: Intensity of transmitted light as a function of angle between polarised and plane of polariser of light.

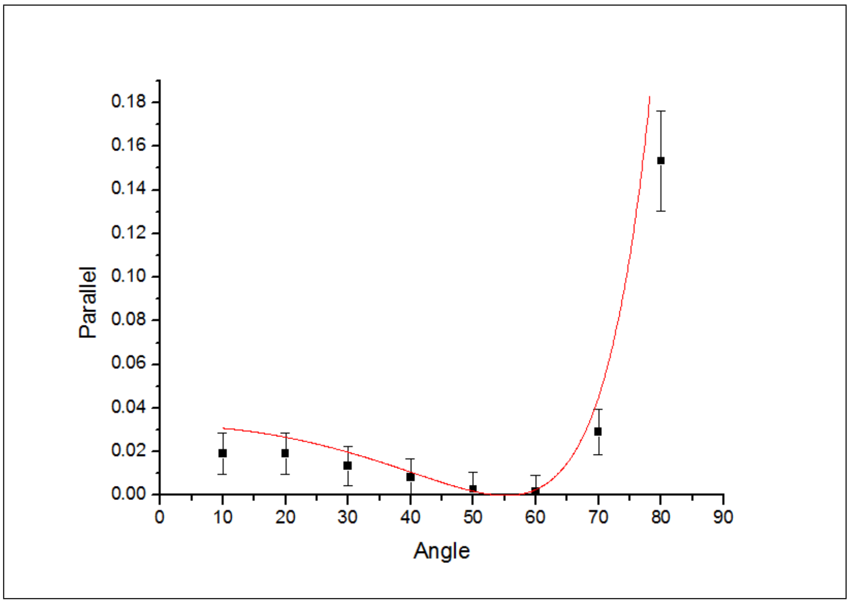

Figure 6: Intensity of reflected light.

The reflectance for both the parallel and perpendicular components of the light are shown in Figure 5. Shown also is a non-linear fit based on the Fresnel formulas which shows the correlation between experimental results and theory.

The p-component is shown alone in Figure 6 where the value for Brewster's angle can be seen more clearly. Rotating the photodiode till a minimum value was found, Brewster's Angle was found to be 59°.

Discussion and Conclusion

- Malus' Law was verified to a high degree of accuracy. Although the initial graph was disappointing, accurately determining the correct 0°relative offset between the laser beam and the polaroid transmission axis after the fact using data analysis allowed us to create a far more accurate graph.

- The experimental data for the reflectance matched the expected curve given by Fresnel's equations, as evidenced by the curve fitted to the data, however the discrepancies were still somewhat high.

- It was found that reflectance increased non-linearly with angle of incidence for light polarised perpendicular to the plane of reflection, while for light polarised parallel to the plane of reflection, there was a minimum at 59°.

- There were several sources of error in this experiment, some more prominent than others.

  The laser took very long (30 minute plus) to stabilise to an acceptable degree. The detector was very sensitive to where exactly the beam fell on it, giving lower results if the beam was not perfectly centred on it. Finally, and most importantly, at the higher angles of incidence it was hard to get the entire image of the beam onto the prism, which meant some of the light was not reflected into the detector.

- The value for Brewster's angle ($59 \pm 0.1°$) led to a value of 1.66 ± 0.05 for the refractive index of the prism which fits with given ranges of refractive indices for glasses.