Gravitational Torsion Pendulum

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Abstract

The aim of this experiment is to measure the gravitational constant G using a torsion pendulum, which we found to be $11.5 \cdot 10^{-11}$ m^{3}N^{-1}s^{-2} which was almost double the accepted value of $6.67 \cdot 10^{-11}$ m^{3}N^{-1}s^{-2}. The experimental result is unreliable due to the unreliable nature of the experiment when performed in an environment from which vibrations can not be isolated, as the apparatus is extremely sensitive to any outside disturbance. Therefore the experimental result cannot be taken to be accurate.

Introduction and Theory

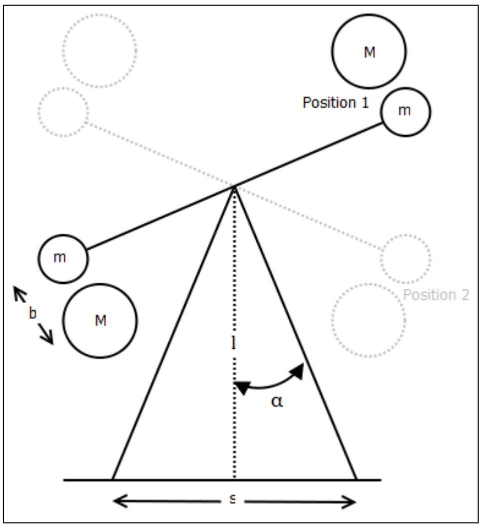

Figure 1: Overhead view of experiment

This experiment is fundamentally similar to that performed by Henry Cavendish in 1798, where a torsion balance is set up and the gravitational attraction between a pair of large spheres and a pair of smaller spheres is measured in terms of the torsion of the wire from which the smaller spheres are suspended from.

In our case, the torsion of the wire is measured by a method whereby a laser beam is reflected from a fixed mirror, attached to the torsion balance, onto a detector some distance (0.67 m) from the apparatus. Thus a small change in the torsion translates to a large change in the position of the reflected beam on the detector.

The apparatus is set up so that the pair of larger lead balls rotate freely about the vertical axis. By rotating the spheres, we can change the direction in which the gravitational attraction between the pairs of spheres causes the wire to twist. The situation is described in Figure 1 (the difference in torsion between the two positions is greatly exaggerated in the diagram).

Then by measuring the difference in torsion between two points, and knowing the restoring force in the wire due to a torsion, and the masses of the larger spheres, and the separations between the balls, it is possible to calculate the gravitational constant, as the following discussion shows.

Derivation of the formula for G

In this experiment, aside from the various masses and distances between objects, as described below, the main measurements to be made were on the period of oscillation, T, of the torsion balance, and the positions of equilibrium, in terms of the deflection angles $\alpha_1 and \alpha_2$, about which the balance oscillated. From these then, can the gravitational constant G be measured. The argument proceeds thus:

When the pendulum is equilibrium in position 1, the gravitational torque, τ_1 due to the attraction between the pairs of spheres is equal to the restoring torque, $k\alpha_1$, of the wire due to the torsion. Thus, $k\alpha_1 = \tau_1$. Similarly, $k\alpha_2 = \tau_2$.

The gravitational attraction between a large sphere and the smaller one is, by Newton's law of gravitation, $F = GMm/b^2$ where b is the separation between the two spheres. Therefore the torque due to both pairs of spheres is

$$\tau = 2\frac{GmM}{b^2}d$$

where d is the distance from the centre of each small sphere to the axis of rotation.

Thus summing the expressions for $k\alpha_1$ and $k\alpha_2$, we have

$$k \cdot (\alpha_1 + \alpha_2) = 4G\frac{mM}{b^2}d$$

The general formula for the period of a harmonic oscillator is

$$T = 2\pi\sqrt{\frac{\text{Inertia}}{\text{Restoring Force}}}$$

thus, for our torsion pendulum we have

$$T^2 = 4\pi^2\frac{I}{k}$$

now the moment of inertia for our pendulum can be modelled as two point masses each a distance d from the axis of our rotation, $I = 2d^2m$, giving us $T^2 = 8\pi^2 d^2 m$, or

$$k = \frac{8\pi^2 d^2 m}{T^2}$$

Thus, we have

$$\frac{8\pi^2 d^2 m}{T^2} \cdot (\alpha_1 + \alpha_2) = 4G\frac{mM}{b^2}d$$

solving for G gives us

$$G = \frac{2\pi^2 b^2 d}{MT^2} \cdot (\alpha_1 + \alpha_2)$$

Finally, since the deflection angles are small, we can see that $\alpha_1 \approxeq tan(\alpha_1) = S_1/L$, thus $\alpha_1 + \alpha_2 = (S_1 + S_2)/L = S/L$, giving us a final formula for G of

$$G = \frac{2\pi^2 b^2 dS}{MT^2 L}$$

Oscillation of the balance

When the lead sphere is in position 1, it performs oscillations, in the plane of all the masses, about a particular equilibrium position α_1. While in theory, the experiment can be left long enough for the oscillations to effectively decay completely, in reality this isn't possible due to the eventuality of outside forces affecting the experiment.

Any force affecting the motion of the masses will result in an unwanted oscillation of the torsion balance, in any of several modes of oscillation, leading to For this reason it was necessary to fit the relevant parts of the actual data to a general damped harmonic oscillator function, $y = Ae^{-\gamma t}\cos(\omega t - \alpha)$

Experimental Method

The lead spheres were placed in position 1, so that the small lead balls oscillate about equilibrium position α_1. The data for the oscillations from the infrared detector were recorded for several full periods of oscillations of the pendulum, i.e. over a time frame from about half an hour to an hour.

Then, the lead spheres were rotated slowly to position 2, and again data from the detector were recorded.

Finally, the equilibrium points and the period of oscillation were measured from the data.

Results and Analysis

- b = Distance between the centres of mass = 0.047 m;
- d = Distance of the midpoints of the lead balls from the axis = 0.045 m
- M = Mass of large sphere = (1.500 ± 0.005) kg;
- L = Distance between the mirror and the detector = 0.67 m

The two plots show the experimental data that was used, along with a curve fitted to match the data. Because the experiment never settled to equilibrium, it was necessary to use nonlinear regression to fit a decaying exponential curve to the parts of data that were usable.

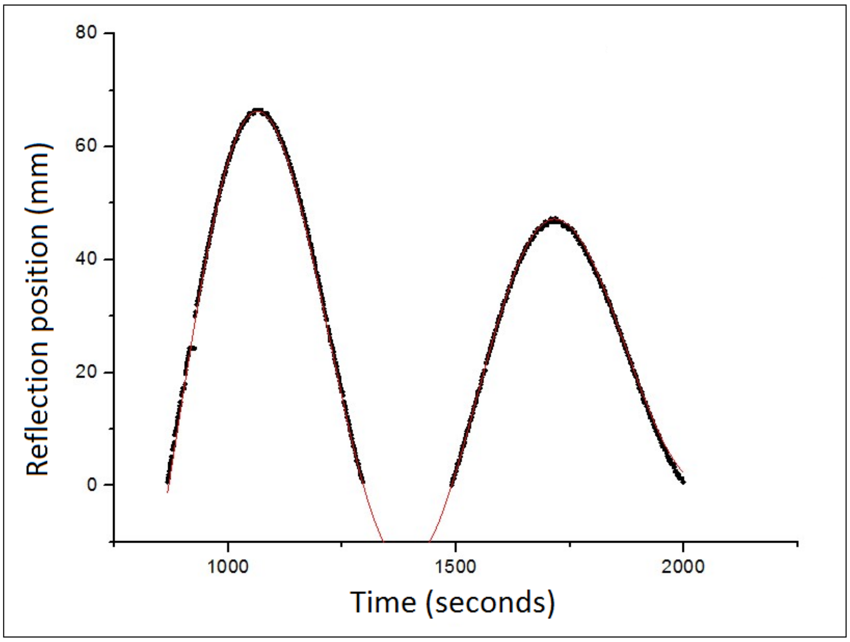

Figure 2: Graph for Position 1

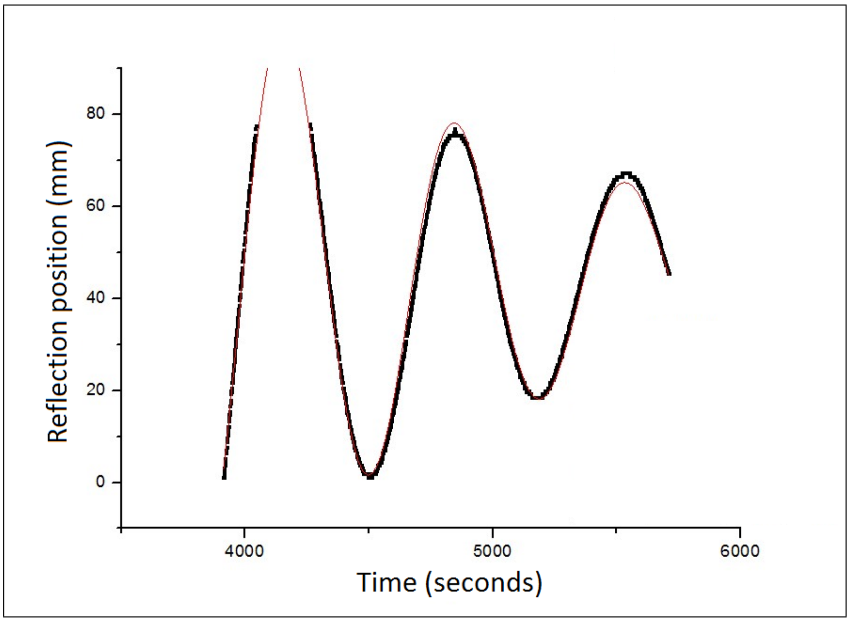

Figure 3: Graph for Position 2

Graph 1 and 2 show the raw data points used along with the curve fitted to the data for Positions 1 and 2 of the experiment respectively, while tables 1 and 2 show the corresponding values for T, S_1 and S_2 computed from the nonlinear curve fit.

From the fit, we found $T = 671$ s and $S = 0.024$ m. Inserting these values into the formula for G gave a final value of $11.5 \cdot 10^{-11}$ $\mathrm{m^3N^{-1}s^{-2}}$ which is almost double the accepted value of $6.67 \cdot 10^{-11}$ $\mathrm{m^3N^{-1}s^{-2}}$.

Model	GPend (User)		
Equation	y0 + A*exp(-((t-t0)/tau))*cos(2*Pi*n*t + f)		
Reduced Chi-Sqr	0.52364		
Adj. R-Square	0.99859		
		Value	Standard Error
B	y0	20.68891	0.03092
B	A	224.37331	--
B	tau	1204.15324	4.7852
B	t0	-847.6088	--
B	n	0.00153	4.4694E-7
B	f	2.22576	0.00357

Figure 4: Table 1

Model	GPend (User)		
Equation	y0 + A*exp(-((t-t0)/tau))*cos(2*Pi*n*t + f)		
Reduced Chi-Sqr	4.22478		
Adj. R-Square	0.99161		
		Value	Standard Error
B	y0	44.52021	0.05428
B	A	129.9375	--
B	tau	1419.0921	12.13521
B	t0	2931.97988	--
B		0.00145	6.33176E-7
B	f	112.84786	0.01874

Figure 5: Table 2

Discussion and Conclusion

The general theme to the entire experiment was the difficulty in getting a set of usable results without the apparatus being disturbed. The entire run for the first week did not result in any usable results, while the results from the second week were only usable thanks to the help from Prof. Stamenov with using nonlinear regression to fit a curve to a subset of data which hadn't been contaminated (detected by looking for unexpected inflections in the curve).

After fitting the curve to the data, it appeared as if the actual data was going to be quite accurate, however the final result was disappointingly far off from the accepted value.