# The Cornu Effect

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November 13, 2011

## Abstract

In this experiment the bulk properties of perspex were investigated, specifically determining values for Young's Modulus and Poisson's ratio, found to be $(2.71\pm0.1)$ GPa and $0.41\pm0.02$ respectively. The two values were determined by measurements on the interference pattern formed by the reflection of monochromatic light from the top of the perspex beam and from the bottom of a glass plate placed on top of the perspex beam, while the perspex beam, placed centrally on two knife edges a short distance apart, had weights suspended from either end.


## Introduction and Theory

The aim of this experiment was to determine, using a method of interference (Newton's Rings), Young's Modulus and Poisson's ratio for perspex.

### Young's Modulus

Suppose we have a block of some material, with a force applied to the ends of the block. In that case, the *stress* is defined as the force per unit area on the block. This force results in a deformation of the material, which is quantified by the *strain*, which is defined as the change per unit length of the block, i.e. the ratio of the change in length to the original length.

This leads us to the definition of *Young's Modulus*, which is defined for some material to be the ratio of stress to strain,

$$Y = \frac{\text{stress}}{\text{strain}}$$

which can, in crude terms, be seen to describe the stiffness of the material, i.e. a high value of Young's Modulus corresponds to a stiff material, e.g. steel, while a low value corresponds to an elastic one, e.g. rubber.

Suppose we have some masses bending a beam, as per our experiment, then the beam is compressed below, and extended above the neutral surface (see Figure 1), thus the beam will be in equilibrium when the restoring forces for these stresses are equal to the forces produced by the load. The internal bending moment is given by

$$\frac{YAk^2}{R_1}$$

where A is the cross-sectional area of beam and k is the radius of gyration (which is $b/\sqrt{12}$ for our beam, where b is the thickness of the beam). Thus we have at equilibrium

$$mgl = \frac{YAk^2}{R_1}$$

or rearranging,

$$Y = \frac{mglR_1}{Ak^2}$$

### Poisson's Ratio

*Poisson's ratio* is defined as the ratio of lateral strain to longitudinal strain

$$\sigma = \frac{\text{lateral strain}}{\text{longitudinal strain}}$$

### Newton's Rings

Newton's rings are the interference patterns seen when a spherical surface is placed on an adjacent flat surface, or more generally the patterns seen from two adjacent surfaces where the distance between the two surfaces varies.

The pattern is caused by interference of the light reflected from the bottom of the top surface with the light from the top of the bottom surface. Light fringes are observed where there is constructive interference, and dark fringes are observed where there is destructive interference. Fringes trace out contours of equal distance between the two surfaces.

If the vertical distance between the two surfaces at some point is one wavelength greater or lesser than the distance at some other point, then a light and dark fringe will be seen between the two points. Because of this, the interference patterns provide an effective way of quantifying the variation of the distance between two surfaces when the variation is small, on the scale of the wavelength of light, across a macroscopic area.

In this experiment then, Newton's Rings are used to estimate the curvature of the perspex beam in both directions.

## Experimental Method

### Experimental Setup

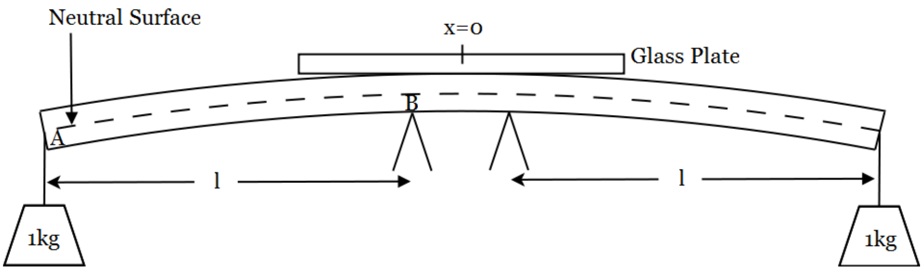


Figure 1: Experimental setup for Cornu Method experiment

The experiment was setup as in Figure 1. We wish to find out what distance The theory states that when the beam is loaded as shown in the experimental setup, the beam is bent into an arc with longitudinal radius of curvature $R_1$. Furthermore, the cross section is also bent into an arc in the opposite direction, with a transverse radius of curvature $R_2$.

From Figure 2 and from using Pythagoras' theorem, we have the formula $(R_1 - f)^2 + x^2 = {R_1}^2$, where $f = d - d_0$. Expanding and rearranging, we have $f^2 - 2R_1 f + x^2 = 0$, solving for f (and discarding the larger solution) gives us

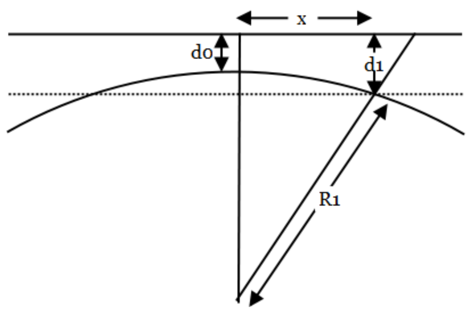


Figure 2: Longitudinal curvature of beam

$f = R_1 - \sqrt{R_1^2 - 4x^2}$. Assuming $x \ll R$, we can use the Taylor series and discard the later terms to get $f = R_1 - (R_1 - \frac{1}{2}\frac{x^2}{\sqrt{R_1^2}}) = \frac{x^2}{2R_1}$. Thus it can be seen that the distance $d_1$ of the perspex beam from the glass plate at a given distance $x$ along the x-axis from the centre is given by the equation

$$d_1 = d_0 + \frac{x^2}{2R_1}$$

Similar reasoning tells us that for the case where we are a given distance $y$ along the y-axis from the centre, the distance $d_2$ is given by the formula

$$d_2 = -\frac{y^2}{2R_2}$$

(since the curvature is in the opposite direction and we have already taken account of $d_0$). Combining the two formulae, we have, for any point $(x, y)$ near the centre

$$d = d_1 + d_2 = d_0 + \frac{x^2}{2R_1} - \frac{y^2}{2R_2}$$

Rearranging, we have

$$\frac{x^2}{R_1} - \frac{y^2}{R_2} = 2(d - d_0)$$

which shows that near the centre, the beam will have the form of a hyperbolic paraboloid. Thus points of constant d will describe hyperbolae, and the interference pattern seen through the microscope will be of concentric hyperbolae, as in Figure 3.

The asymptotes of the hyperbolae are given by $x^2R_2 = y^2R_1$, i.e. the asymptotes are a pair of lines each making an angle $\theta$ with the x-axis, which means that

$$\text{Cot}^2\theta = \frac{R_1}{R_2} = \sigma$$

which allows us to calculate Poisson's ratio for perspex with either the ratio of curvature or the angle formed by the asymptotes.

## Procedure (Part 1)

The first part of the experiment required finding the relation between the horizontal displacement and vertical displacement, along both the $x$ and $y$ axes, by measuring the distance travelled by the microscope as it counted rings.

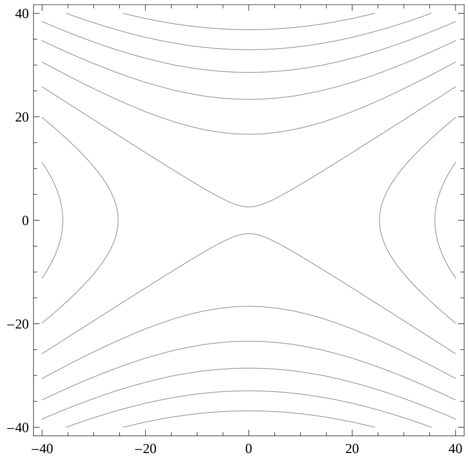


Figure 3: Concentric hyperbolae as would be seen through microscope

1. Set up apparatus as in Figure 1
2. Turn on mercury lamp
3. Bring travelling microscope over centre of perspex beam so fringes are observed.
4. Arrange the apparatus so that the travelling microscope moves over the beam along the x-axis (i.e. parallel to the length of the beam).
5. Starting from the centre, record the distance of each successive dark ring from the centre for around 10 rings.
6. Repeat along the y-axis (i.e. across the beam).

## Convex Lens pattern

At this point the experiment was repeated with a convex lens, and a pattern of concentric ellipses was observed through the microscope. This pattern can be explained thus. The bottom surface of a convex lens is a portion of the surface of some sphere, i.e. the loci of points of constant distance $f_1$ from some point above lens. This means that we can derive the vertical distance from the lens to the horizontal plane in the exact same manner as we did with the perspex beam. Thus, the formula for the points of constant vertical distance between the perspex beam and the lens is

$$\underbrace{\frac{x^2}{f_1} + \frac{y^2}{f_1}}_{\text{lens curvature}} \underbrace{\frac{x^2}{2R_1} - \frac{y^2}{2R_2}}_{\text{beam curvature}} = 2(d - d_0)$$

Rearranging, we get

$$\frac{(f_1 + R_1)x^2}{2R_1 f_1} + \frac{(f_1 - R_2)y^2}{2R_2 f_1} = 2(d - d_0)$$

which is the general equation for a paraboloid. However though, in general $f_1 > R_2$ and the second term is positive, giving an elliptical paraboloid, and hence the contours are a series of concentric ellipses (Figure 4), as seen as in the intereference pattern. Interestingly, at some critical deformation of the beam, the second term will cancel out completely, which would give rise to an interference pattern of straight lines.

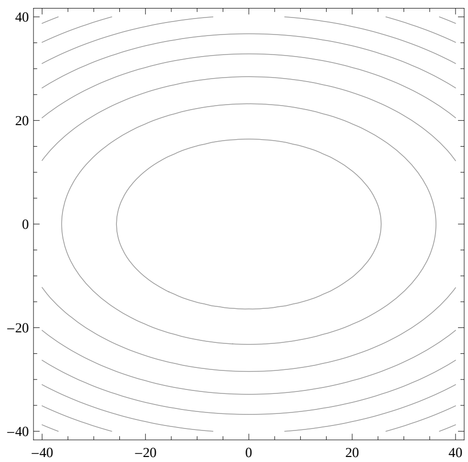


Figure 4: Concentric ellipses as would be seen through microscope

# Results and Analysis

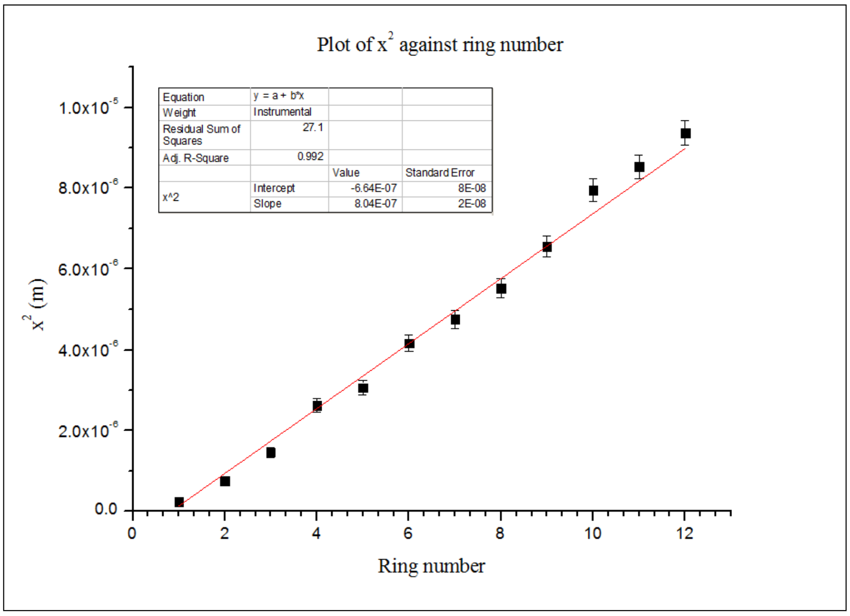


Figure 5: Plot of $x^2$ against ring number

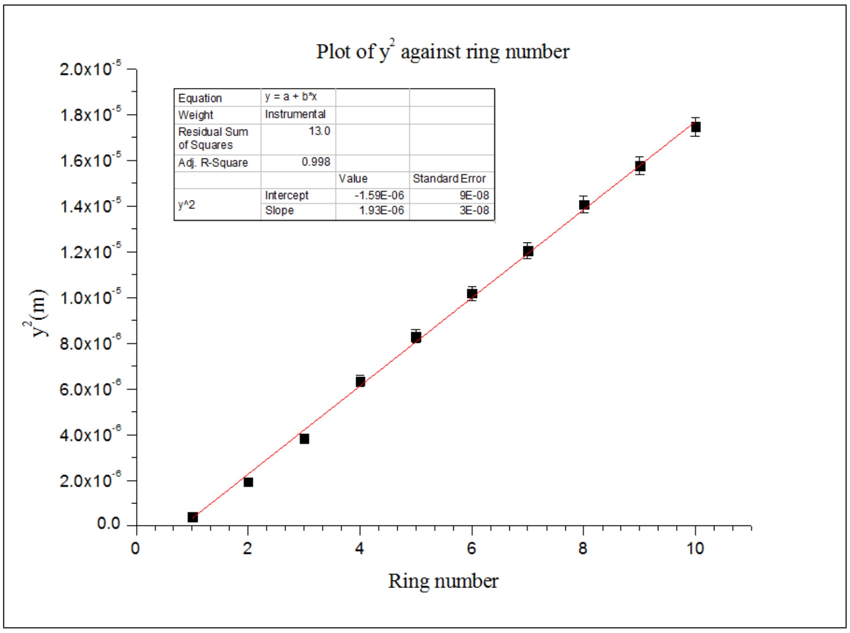


Figure 6: Plot of $y^2$ against ring number

The experimental results for the travelling microscope experiments are shown in Figure 5 and Figure 6, showing $x^2$ and $y^2$ against ring number respectively, where $x$ and $y$ are the distances from the centre of the beam of each successive dark ring.

The length $l$ of the end of the beam to the knife edges was found to be

$$l = 0.150 \pm 0.001\text{m}$$

Measurement of the perspex beam found the cross-sectional height b and the cross-sectional width a to be 0.006±0.0002mm and 0.041±0.0002m respectively. This gives a cross-sectional area

$$A = 2.46 \pm 0.08x10^{-4}\text{m}^2$$

While keeping the y displacement constant at 0, the formula relating $x, R_1$ and ring number $N$ is $x^2 = R_1(N\lambda - 2d_0)$. Therefore the slope of the graph is $N\lambda$, where $\lambda = 589.3x10^{-6}$m is the wavelength of sodium light. Since the slope is $8.04x10^{-7} \pm 2x10^{-8}$m, this gives us

$$R_1 = 1.36 \pm 0.03\text{m}$$

Similarly, we have

$$R_2 = 3.28 \pm 0.03\text{m}$$

Measuring the angle of the asymptotes with the protractor gives a value of

$$\theta = 55\,\text{deg} \pm 5\,\text{deg}$$

## Discussion and Conclusions

Taking the ratio of $R_1$ to $R_2$ gives us a value of

$$\sigma = 0.41 \pm 0.02$$

This corresponds to quoted values of 0.39 for Poisson's ratio.

Using the crude method of measuring the angle of the asymptotes with the protractor, we get a value of $0.49 \pm 0.16$ for Poisson's ratio.

Using the formula $Y = \frac{mglR_1}{Ak^2}$ for Young's Modulus, we get

$$Y = (2.71 \pm 0.1) \cdot 10^9 \mathrm{P}a$$

which corresponds to quoted values of 0.27 - 0.35 GPa for Young's modulus.

Overall, the experiment was successful, as both values corresponded to values quoted in the literature.

## References


1. `http://www.roymech.co.uk/Useful_Tables/Matter/Prop_Solids.htm` (Value for Young's Modulus.)
2. `http://www.theplasticshop.co.uk/plastic_technical_data_sheets/working_with_perspex_manual.pdf` (Value for Poisson's Ratio.)