UNIVERSITY OF DUBLIN

TRINITY COLLEGE

FACULTY OF ENGINEERING, MATHEMATICS & SCIENCE

SCHOOL OF PHYSICS

Senior Freshman
Annual Examination

Trinity Term 2012

Classical Physics

(Sciences (Physics), Chemistry with Molecular Modelling, Physics and Chemistry of Advanced Materials, Theoretical Physics,)

Tuesday 8th May 2012 | RDS Main | 09.30 – 12.30

Professors J. McGilp, E. McCabe, G. Cross, H. Zhang, C. Patterson

ALL QUESTIONS CARRY EQUAL MARKS.

USE SEPARATE ANSWER BOOKS FOR EACH SECTION

Both old log tables (Mathematics Tables) and new log tables (Booklet of Formulae and Tables) are available from the invigilator for all students who require them. Graph paper is also available.

Non-programmable calculators are permitted for this examination – please indicate the make and model of your calculator on each answer book used.

Science, Chemistry with Molecular Modelling, Physics & Chemistry of Advanced Materials Students

Answer *FIVE* questions, AT LEAST ONE from each of Sections A, B, C and D AND ONE OTHER from these sections, in 3 hours.

Theoretical Physics Students

Answer *FIVE* questions, AT LEAST ONE from each of Sections A, B, C and E AND ONE OTHER from these sections, in 3 hours.

SECTION A

1. A mass m of 0.2 kg is attached to a weightless spring that is anchored to a support. The mass oscillates at a frequency f of 6.0 Hz when it is displaced and released.

   (a) Find the spring constant k in the appropriate SI units

   [2 marks]

   (b) Find the maximum acceleration of the oscillation if the mass is displaced from its initial position by 0.3 m and released.

   [3 marks]

   (c) The amplitude of the oscillation decreases to 0.06 m in 8.0 s. Find the resistance b of the system in the appropriate SI units.

   [5 marks]

$\omega_0^2 = \sqrt{\frac{k}{m}}$

$kx_0 = mg$

$\omega = 2\pi f$

$T = 1/f$

$m\ddot{x} - kx - b\dot{x}$

$\Rightarrow x = A\cos(\omega t + \phi)$

$\omega_0 = \sqrt{\frac{k}{m}}$

$\omega_0^2 = \frac{k}{m}$

$\Rightarrow k = \omega_0^2 m$

$= \text{kg rad/s}$

$k = \frac{\pi}{6(0.5)} = \frac{\text{rad}}{m}$

2. An elastic cord strung with N beads of mass m and separation a has discrete transverse vibrational modes, the n th mode frequency of which is given by

$$\omega_n = 2\,\omega_0 \sin\left\{\frac{n\pi}{2(N+1)}\right\}$$

where $\omega_0 = \sqrt{T_0/ma}$ and T_0 is the tension in the cord.

For such a system with 5 beads:

(a) state the number of independent modes of vibration of the system and explain your statement;

[1 mark]

(b) find the mode frequencies ω_n, if $\omega_0 = 15 \text{ rad s}^{-1}$;

[1 mark]

(c) find the wave numbers of the modes k_n, if $a = 0.5\,\text{m}$;

[3 marks]

(d) sketch the frequency behaviour of both the beaded cord and an unbeaded elastic cord, as a function of k, labeling your axes carefully and marking key points on the axes;

[3 marks]

(e) discuss the physics underlying the different behaviour of the beaded cord and the unbeaded cord.

[2 marks]

SECTION B

3. (a) Consider the wave

$\Psi(y,t)=15\mathrm{Sin}(4y+20t)$

Explain briefly why this is a travelling wave. Calculate the wave's amplitude, angular frequency, period and the speed at which it is travelling.

[3 marks]

(b) Show that the wave in (a) satisfies the differential wave equation.

[2 marks]

(c) Write the expression for the total energy density associated with a light wave. Hence show how to arrive at an expression for the Poynting vector.

[2 marks]

(d) Using part (c) show how to arrive at an expression for the light irradiance.

[1 mark]

(e) Consider an electric field of 200V/m normally incident on a domestic cat. Using an appropriate expression, estimate the rate of energy absorption by the cat.

[2 marks]

$u_E = \frac{\varepsilon}{2} E$

$\frac{1}{2\mu} B$

$c = EB$

4. (a) Explain what is meant by birefringence.

[1 mark]

(b) Describe the operation of a Glan-Foucault polariser. Explain in detail how the refractive index of the gap material and the prism angle influence this operation

[3 marks]

(c) If the possible range of prism angles for the Glan-Foucault arrangement to work as a polariser are 36.9°<ϕ<42.1° calculate the corresponding relevant refractive indices of the prism material.

[2 marks]

(d) Outline the difference between the Glan-Foucalt polariser and the Glan-Thompson polariser.

[1 mark]

(e) By using appropriate diagrams, explain why when scattered light is observed perpendicular to the original propagation direction, it is completely linearly polarised.

([3 marks])

$n_o < \theta < n_e$

SECTION C

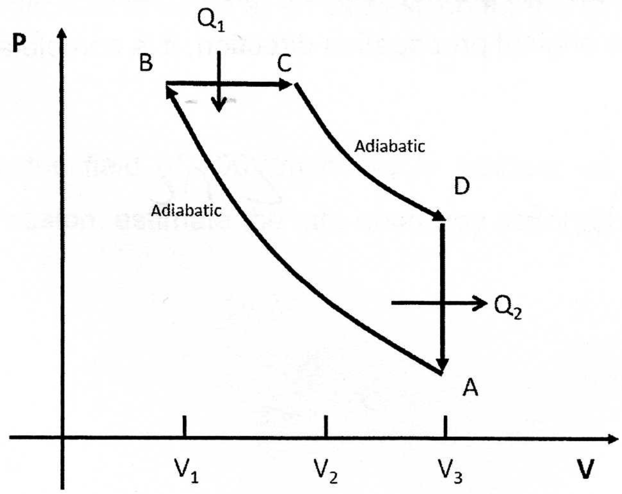
5. (a) For an ideal gas, show that in a reversible, adiabatic process $PV^{\gamma} = const$ where $\gamma = C_P / C_V$ is the ratio of heat capacity at constant pressure and constant volume for the gas, respectively. You can assume that $C_P - C_V = nR$.

[5 marks]

(b) Diesel Engine. In class and homework, we analysed the efficiency of various engines in detail. Show that the efficiency of the Diesel engine can be expressed in terms of its compression ratio $r_c = V_3/V_1$ and expansion ratio $r_e = V_3/V_2$ as follows

$$\eta = 1 - \frac{1}{\gamma}\left[\frac{\dfrac{1}{r_e^{\gamma}} - \dfrac{1}{r_c^{\gamma}}}{\dfrac{1}{r_e} - \dfrac{1}{r_c}}\right]$$

where the working substance is assumed to be an ideal gas and the Diesel cycle is given:



[5 marks]

U = Q+W

H = U + VdP

U = U(t)

dU = dQr + PdV

H = U + PdV

Cp = dQ/dT

dU = dQr + PdV

6. The equation of state for a rubber band at temperature T stretched to length L is

$$\mathcal{F} = aT\left[\frac{L}{L_0} - \left(\frac{L_0}{L}\right)^2\right]$$

Where $\mathcal{F}$ is the tension in the rubber band, $L_0 = 1m$ the unstretched length, and $a = 0.02\,NK^{-1}$ is a constant.

(a) Write the Central Equation for the rubber band, remembering that stretching longer means positive work done to the system in this case

[1 mark]

(b) Derive the energy equation $\left(\frac{\partial U}{\partial L}\right)_T$ for the rubber band (Hint: You can use the equivalent Maxwell relations for this system, in particular replace P with $-\mathcal{F}$ to account for the change in sign for positive work):

[3 marks]

(c) Show that U is a function of T only, like we found for the ideal gas

[2 marks]

(d) What is the work done for an isothermal, reversible stretch of the elastic band from 1 m to 2 m at $T = 300\,K$?

[2 marks]

(e) If the heat capacity $C_L = 1.0\,JK^{-1}$, what is the final temperature for an isentropic stretch from 1 m to 2m starting at $T = 300\,K$? (Hint, start from the appropriate TdS law for this $\mathcal{F}dL$ system.)

[2 marks]

SECTION D

7. The circuit shown in Figure 7 is an example which describes the general nature of circuit response to sinusoidal sources. There, v_s is a sinusoidal voltage i.e. $v_s = V_m \cos(\omega t + \phi)$. For convenience, we assume the initial current in the circuit to be zero and measure time from the moment the switch is closed.

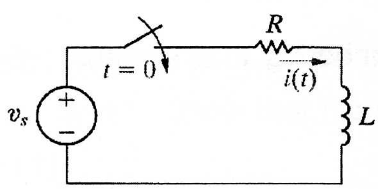


Figure 7

(a) Derive the equation that describes the behaviour of the current $i(t)$ when $t \geq 0$.

[2 marks]

(b) The solution to the equation has two components: transient (i_t) and steady-state (i_s) responses. It can be written as $i(t) = i_t(t) + i_s(t)$. Construct the frequency-domain equivalent circuit and find the steady-state response $i_s(t)$.

[5 marks]

(c) The transient response is given by

$$i_s(t) = -\frac{V_m}{\sqrt{R^2+\omega^2 L^2}} \cos(\phi - \theta) \exp\left(-\frac{t}{\tau}\right) .$$

Determine the time constant τ and check the validity of the total response $i(t) = i_t(t) + i_s(t)$

[3 marks]

8. Circuit analysis plays an important role in the analysis of systems designed to transfer power from a source to a load. Maximum power transfer can best be described with the aid of the circuit shown in Figure 8a.

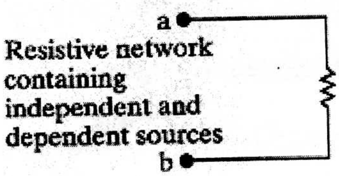


Figure 8a

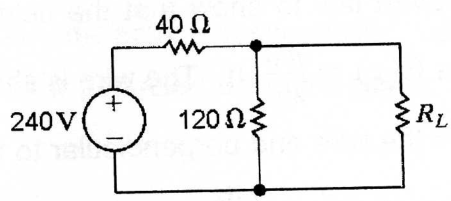


Figure 8b

(a) A resistive network can always be replaced by its Thévenin equivalent. Replace the network by its Thévenin equivalent and redraw the circuit.

[2 marks]

(b) Determine the value of the Thévenin resistance that permits maximum power delivery to R_L

[3 marks]

(c) For the circuit shown in Figure 8b, find the value of R_L that results in maximum power being transferred to R_L

[3 marks]

(d) For the circuit shown in Figure 8b, calculate the maximum power that can be delivered to R_L. When R_L is adjusted for maximum power transfer, what percentage of the power delivered by the 360V source reaches R_L

[2 marks]

SECTION E

9. The Biot-Savart law states that the magnetic induction, $d\mathbf{B}(\mathbf{r})$, at $\mathbf{r}$ due to a current element $d\boldsymbol{\ell}$ at $\mathbf{r}'$ is $d\mathbf{B}(\mathbf{r}) = \frac{\mu_0 I}{4\pi} \frac{d\boldsymbol{\ell} \times (\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3}$.

(a) Use the Biot-Savart law to show that the field outside an infinite, thin wire carrying a current, I, at $\mathbf{r}$ is $\mathbf{B}(\mathbf{r}) = \frac{\mu_0 I}{2\pi r} \mathbf{n}$. The wire is shown in the figure below. $\mathbf{n}$ is a unit vector perpendicular to the wire and perpendicular to the plane of the paper in the figure. You will need the integral $\int_{-\infty}^{\infty} \frac{r\, dz}{(r^2+z^2)^{3/2}} = \frac{2}{r}$.

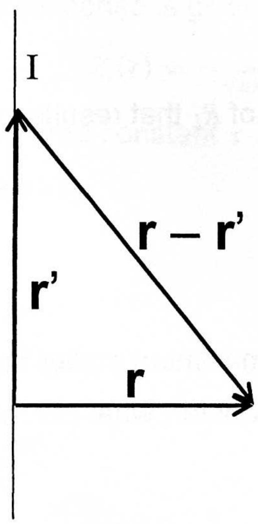
[5 marks]

(b) State Ampère's law in its integral and differential forms in words and equations. Show that one form may be obtained from the other by applying Stokes' theorem.

[3 marks]

(c) Show that the field in part (a) satisfies the integral form of Ampere's law by evaluating the line integral of $\mathbf{B}(\mathbf{r})$ in a closed circle which is perpendicular to the wire.

[2 marks]



10. (a) State Gauss' law precisely in its integral and differential forms, in words and as equations.

[2 marks]

(b) Explain the terms: polarisation, dielectric susceptibility, depolarising field and macroscopic field as they apply to a polarised dielectric block. Give the S.I. units for each of these.

[4 marks]

(c) Use Gauss' law in its integral form to show that the depolarising field in a dielectric block is -**P** / ε_o, where **P** is the polarisation. Using your answer to part (b), find an expression for the relationship between the macroscopic field inside the block and the applied field which polarises the block.

[4 marks]

TRINITY COLLEGE DUBLIN
SCHOOL OF PHYSICS

Electron rest mass	m_e	9.11×10^{-31} kg
Proton rest mass	M_p	1.67×10^{-27} kg
Electronic charge	e	1.60×10^{-19} C
Speed of light in free space	c	3.00×10^{8} m s^{-1}
Planck's constant	h	6.63×10^{-34} J s
	$h/2\pi = \hbar$	1.05×10^{-34} J s
Boltzmann's constant	k	1.38×10^{-23} J K^{-1}
Molar gas constant	R	8.31×10^{3} JK^{-1}kmol^{-1}
Avogadro's number	N_A	6.02×10^{26} kmol^{-1} $= 6.02 \times 10^{23}$ mol^{-1}
Standard molar volume		22.4×10^{-3} m^3
Bohr magneton	μ_B	9.27×10^{-24} A m^2 OR J T^{-1}
Nuclear magneton	μ_N	5.05×10^{-27} A m^2 OR J T^{-1}
Bohr radius	a_o	5.29×10^{-11} m
Fine structure constant $e^2/(4\pi\varepsilon_o\hbar c)$	$= \alpha$	$(137)^{-1}$
Rydberg's constant	R_∞	1.10×10^{7} m^{-1}
Stefan's constant	σ	5.67×10^{-8} W m^{-2} K^{-4}
Gravitational constant	G	6.67×10^{-11} N m^2 kg^{-2}
Proton magnetic moment	μ_p	2.79 μ_N
Neutron magnetic moment	μ_n	-1.91 μ_N
Permeability of free space	μ_o	$4\pi \times 10^{-7}$ H m^{-1}
Permittivity of free space	ε_o	8.85×10^{-12} F m^{-1}
1 electron volt	eV	1.60×10^{-19} J
1 unified atomic mass unit (^{12}C scale)		1.66×10^{-27} kg = 931 MeV/c^2
Wavelength of 1 eV photon		1.24×10^{-6} m
1 atmosphere		1.01×10^{5} N m^{-2}
Standard acceleration due to gravity		10 m s^{-2}
Free space impedance Z_o		377 Ω

Astronomical unit	au	1.50×10^{11} m
Parsec	pc	3.09×10^{16} m
Solar radius	$R_\odot$	6.96×10^{8} m
Solar mass	$M_\odot$	1.99×10^{30} kg
Solar luminosity	$L_\odot$	3.85×10^{26} W
Earth mass	$M_\oplus$	5.97×10^{24} kg
Earth radius (equatorial)	$R_\oplus$	6378 km