I rewrite $\vec{E}$ with all $\beta\cdot\vec{\beta}$ removed (since $\beta\cdot\vec{\beta}=0$):

Let $a = \gamma c R(1-\vec{\beta}\cdot\vec{n})$

$$\vec{E} = -\frac{e}{a}\left\{\frac{Rc\gamma^2\dot{\vec{\beta}}}{a} - \frac{R\gamma c\dot{\vec{\beta}} - R\vec{n}\gamma c}{a^2}\left[-c^2 - Rc\gamma^2\vec{n}\cdot\dot{\vec{\beta}}\right]\right\}_{\tau=\tau_0}$$

$$= -\frac{e}{a}\left\{\frac{Rc\gamma^2\dot{\vec{\beta}}}{a} + \frac{R\gamma c^3\dot{\vec{\beta}}}{a^2} - \frac{R\gamma c^3\vec{n}}{a^2} + \frac{R^2\gamma^3c^2\dot{\vec{\beta}}(\vec{n}\cdot\dot{\vec{\beta}})}{a^2} - \frac{R^2\gamma^3c^2\vec{n}(\vec{n}\cdot\dot{\vec{\beta}})}{a^2}\right\}_{\tau=\tau_0}$$

$$= -\frac{Rc\gamma e}{a^2}\left\{\frac{\gamma\dot{\vec{\beta}}}{1} + \frac{c^2\dot{\vec{\beta}}}{a} - \frac{c^2\vec{n}}{a} + \frac{R\gamma^2c\,\dot{\vec{\beta}}(\vec{n}\cdot\dot{\vec{\beta}})}{a} - \frac{R\gamma^2c\,\vec{n}(\vec{n}\cdot\dot{\vec{\beta}})}{a}\right\}_{\tau=\tau_0}$$

$$= -\frac{Rc\gamma e}{a^2}\left\{\frac{\gamma\dot{\vec{\beta}}}{1} + \frac{c^2\dot{\vec{\beta}}}{a} - \frac{c^2\vec{n}}{a} + \frac{R\gamma^2c}{a}(\dot{\vec{\beta}}-\vec{n})(\vec{n}\cdot\dot{\vec{\beta}})\right\}_{\tau=\tau_0}$$

$$= \frac{-Rc\gamma e}{a^2}\left\{\frac{\gamma\dot{\vec{\beta}}}{1} + \frac{c^2(\dot{\vec{\beta}}-\vec{n})}{a} + \frac{R\gamma^2c}{a}(\dot{\vec{\beta}}-\vec{n})(\vec{n}\cdot\dot{\vec{\beta}})\right\}_{\tau=\tau_0}$$

$$= \frac{Rc\gamma e}{a^2}\left\{\frac{c^2(\vec{n}-\dot{\vec{\beta}})}{a} - \frac{\gamma\dot{\vec{\beta}}}{1} + \frac{R\gamma^2c}{a}(\vec{n}-\dot{\vec{\beta}})(\vec{n}\cdot\dot{\vec{\beta}})\right\}_{\tau=\tau_0}$$

$$= \frac{Rc\gamma e}{a^2}\left\{\frac{c^2(\vec{n}-\dot{\vec{\beta}})}{a} - \frac{R\gamma^2c\,\dot{\vec{\beta}}(1-\vec{\beta}\cdot\vec{n})}{a} + \frac{R\gamma^2c}{a}(\vec{n}-\dot{\vec{\beta}})(\vec{n}\cdot\dot{\vec{\beta}})\right\}_{\tau=\tau_0}$$

$$= \frac{Rc\gamma e}{a^2}\left\{\frac{c^2(\vec{n}-\dot{\vec{\beta}})}{a} + \frac{R\gamma^2c}{a}\left[(\vec{n}-\dot{\vec{\beta}})(\vec{n}\cdot\dot{\vec{\beta}}) - \dot{\vec{\beta}}(1-\vec{\beta}\cdot\vec{n})\right]\right\}_{\tau=\tau_0}$$

$$= \left.\frac{Rc^3\gamma e(\vec{n}-\vec{\beta})}{\gamma^3c^3R^3(1-\vec{\beta}\cdot\vec{n})^3}\right|_{\tau=\tau_0} + \left.\frac{R^2\gamma^2c^2e}{\gamma^3c^3R^3\cdot(1-\vec{\beta}\cdot\vec{n})^3}\left[(\vec{n}-\dot{\vec{\beta}})(\vec{n}\cdot\dot{\vec{\beta}}) - \dot{\vec{\beta}}(1-\vec{\beta}\cdot\vec{n})\right]\right|_{\tau=\tau_0}$$

see vector identity overleaf

$$= \left[\frac{e}{\ldots}\frac{(\vec{n}-\vec{\beta})}{\ldots}\right] + \left[\frac{e}{\ldots}\left(\vec{n}\times\left[(\vec{n}-\vec{\beta})\times\dot{\vec{\beta}}\right]\right)\right]$$

We use the vector identity $A\times(B\times C) = B(A\cdot C) - C(A\cdot B)$

$$\vec{n}\times\left(\left(\vec{n}-\vec{\beta}\right)\times\dot{\vec{\beta}}\right) =$$

$$\boxed{\begin{array}{l} A = \vec{n} \\ B = \vec{n}-\vec{\beta} \\ C = \dot{\vec{\beta}} \end{array}}$$

$$= \left(\vec{n}-\vec{\beta}\right)\left(\vec{n}\cdot\dot{\vec{\beta}}\right) - \dot{\vec{\beta}}\left[\vec{n}\cdot\left(\vec{n}-\vec{\beta}\right)\right]$$

$$= \left(\vec{n}-\vec{\beta}\right)\left(\vec{n}\cdot\dot{\vec{\beta}}\right) - \dot{\vec{\beta}}\left(1-\vec{n}\cdot\vec{\beta}\right)$$