MA1E01: Solutions week 6

Solution 1 *The function is the composition of* $f_2(x) = \sqrt{x}$ *and* $f_1(x) = 3x^4 + 12x^2 + 1$, *so applying the chain rule to* $f(x) = f_2(f_1(x))$ *we get*

$$f_2'(f_1(x)) = \frac{1}{2\sqrt{3x^4 + 12x^2 + 1}}$$

and

$$f_1'(x) = 12x^3 + 24x\,,$$

so

$$f'(x) = \frac{6x^3 + 12x}{\sqrt{3x^4 + 12x^2 + 1}}$$

Solution 2 *The limit definition of the derivative at* $x = 1$ *is:*

$$f'(1) = \lim_{h\to 0} \frac{f(1+h) - f(1)}{h}$$

This is a $0/0$ *limit, so we have to be careful...*

$$f'(1) = \lim_{h\to 0} \frac{f(1+h) - f(1)}{h} = \lim_{h\to 0} \frac{(1+h)^2 + 1 - 1 - 1}{3h} = \lim_{h\to 0} \frac{2+h}{3} = 2/3$$

On the other hand, applying the rules of derivation

$$f'(x) = 2x/3 \tag{1}$$

and therefore $f'(1) = 2/3$.

Solution 3 *The function is a ratio of* $r(x) = \sin(3x^4 + 12x^2 + 1)$ *and* $g(x) = x^2 + 1$, *so we get*

$$f'(x) = \frac{r'(x)g(x) - r(x)g'(x)}{g^2(x)}$$

with

$$g'(x) = 2x\,,$$

and $r(x) = r_2(r_1(x))$ *the composition of* $r_2(x) = \sin x$ *with* $r_1(x) = 3x^4 + 12x^2 + 1$, *so*

$$r'(x) = r_2'(r_1(x))\, r_1'(x) = 12(x^3 + 2x)\cos(3x^4 + 12x^2 + 1)\,.$$

So we have

$$f'(x) = \frac{12(x^3 + 2x)\cos(3x^4 + 12x^2 + 1)\,(x^2 + 1) - 2x\sin(3x^4 + 12x^2 + 1)}{(x^2 + 1)^2}$$

Solution 4 *Applying the chain rule*

- $\frac{\mathrm{d}}{\mathrm{d}x}\sqrt{\sin x} = \frac{\cos x}{2\sqrt{\sin x}}$

- $\frac{\mathrm{d}}{\mathrm{d}x}\sqrt{x^3+2x} = \frac{3x^2+2}{2\sqrt{x^3+2x}}$
- $\frac{\mathrm{d}}{\mathrm{d}x}\sqrt{f(x)} = \frac{f'(x)}{2\sqrt{f(x)}} = \frac{1}{2\sqrt{f(x)}}\frac{\mathrm{d}}{\mathrm{d}x}f(x)$
- $\frac{\mathrm{d}}{\mathrm{d}x}\sqrt{y(x)} = \frac{y'(x)}{2\sqrt{y(x)}} = \frac{1}{2\sqrt{y(x)}}\frac{\mathrm{d}}{\mathrm{d}x}y(x)$

Solution 5

- *Taking the derivative with respect to x*

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\sqrt{y}+y\right] = \frac{\mathrm{d}}{\mathrm{d}x}3 \Longrightarrow \left[\frac{1}{2\sqrt{y}}+1\right]\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Longrightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 0$$

  (Not surprising, since y is constant!!!!!!!!!)

- *Taking the derivative with respect to x*

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\sqrt{y}+y\right] = \frac{\mathrm{d}}{\mathrm{d}x}3x \Longrightarrow \left[\frac{1}{2\sqrt{y}}+1\right]\frac{\mathrm{d}y}{\mathrm{d}x} = 3 \Longrightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3}{1+\frac{1}{2\sqrt{y}}}$$

- *Taking the derivative with respect to x*

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\sqrt{y}+\sin y\right] = \frac{\mathrm{d}}{\mathrm{d}x}3x^4 \Longrightarrow \left[\frac{1}{2\sqrt{y}}+\cos y\right]\frac{\mathrm{d}y}{\mathrm{d}x} = 12x^3 \Longrightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{12x^3}{\cos y+\frac{1}{2\sqrt{y}}}$$

Solution 6 *Taking the derivative with respect to x*

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\sqrt{\sin(\cos y)}+x^2\cos y\right] = \frac{\mathrm{d}y}{\mathrm{d}x}$$

Now we use

$$\frac{\mathrm{d}}{\mathrm{d}x}\sqrt{\sin(\cos y)} = \frac{1}{2\sqrt{\sin\cos y}}\left[\cos(\cos y)\right](-\sin y)\frac{\mathrm{d}y}{\mathrm{d}x}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}x^2\cos y = 2x\cos y - x^2\sin y\frac{\mathrm{d}y}{\mathrm{d}x}$$

and therefore

$$\frac{\mathrm{d}y}{\mathrm{d}x}\left[\frac{1}{2\sqrt{\sin\cos y}}\left[\cos(\cos y)\right](-\sin y) - x^2\sin y - 1\right] = -2x\cos y$$

or

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-2x\cos y}{\frac{1}{2\sqrt{\sin\cos y}}\left[\cos(\cos y)\right](-\sin y) - x^2\sin y - 1}$$