

MA1E01: Tutorial week 5

REMEMBER TO HAND BEFORE THE TUTORIAL STARTS

- Limits, continuity and sequences.

Problem 1 Compute the following limits

1. $\lim_{x \rightarrow 0^+} \frac{\sin x}{\sqrt{x}}$
2. $\lim_{x \rightarrow 0^-} \frac{\sin x}{\sqrt{x}}$
3. $\lim_{x \rightarrow 0} \frac{\sin x}{\sqrt{x}}$

Problem 2 Compute the following limits

1. $\lim_{x \rightarrow \infty} \frac{3x+1}{2x^2-2x-4}$
2. $\lim_{x \rightarrow \infty} \frac{3x^4+3x^2+4 \sin x+2}{2x^4+3x^2-2}$
3. $\lim_{x \rightarrow \infty} \frac{3x^2+3x+4 \sin(x^7)-3}{3x^2-2}$

Problem 3 Is the function

$$f(x) = \begin{cases} x \sin(1/x) & x \neq 0 \\ 0 & x = 0 \end{cases}$$

continuous everywhere?

Problem 4 Using that

$$\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e, \quad (1)$$

show that

1. $\lim_{x \rightarrow 0} (1+x)^{1/x} = e$
2. Compute the limit

$$\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x^3 + 2x - 1}\right)^x, \quad (2)$$

3. Compute the limit

$$\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x-1}\right)^{x+1}, \quad (3)$$

4. Compute the limit

$$\lim_{x \rightarrow 0} \left(1 + \frac{1}{x+1}\right)^{x+1}, \quad (4)$$

Problem 5 Consider the sequence defined by the recursive relation

$$a_n = a_{n-1} + a_{n-2}.$$

With $a_1 = 23$ and $a_2 = 122$. Determine

$$\lim_{n \rightarrow \infty} \frac{a_{n-1}}{a_n}.$$

How does the result depend on the initial condition(s) (a_1 and a_2)