MA1E01: Solutions week 3

Solution 1 *From the figure we have the set of equations*

$$\begin{aligned} h-1 &= d\tan 45 = d \\ h-2 &= d\tan 30 = d/\sqrt{3} \end{aligned}$$

Dividing both equations, one obtains

$$\frac{h-1}{h-2} = \sqrt{3} \Longrightarrow h = \frac{2\sqrt{3}-1}{\sqrt{3}-1} \approx 3.4m$$

Solution 2

1. *We solve*

$$y = \sqrt[3]{x^3+5} \Longrightarrow x = \sqrt[3]{y^3-5}\,.$$

Therefore the inverse function is

$$f^{-1}(x) = \sqrt[3]{x^3-5}\,. \qquad (1)$$

2. *We solve*

$$y = \sqrt{x-1} \Longrightarrow x = y^2+1\,.$$

Note that the domain of $f(x)$ *is* $x \geq 1$, *and its range* $f(x) \in [0,\infty)$. *Therefore the domain of* $f^{-1}(y)$ *is* $y \in [0,\infty)$. *We have*

$$f^{-1}(x) = x^2+1, \quad x \geq 0\,.$$

3. *We solve*

$$y = \sqrt{x^2+1} \Longrightarrow x^2 = y^2-1\,.$$

This last equation has two solutions $x = \pm\sqrt{y^2-1}$. *Since the solution is not unique, the inverse does not exist.*

Solution 3

1. *We solve*

$$y = \sqrt[5]{x^3-1} \Longrightarrow x = \sqrt[3]{y^5+1}\,.$$

Therefore the inverse is

$$f^{-1}(x) = \sqrt[3]{x^5+1}\,.$$

2. *We solve*

$$y = \sqrt{x^3-27}-1 \Longrightarrow x = \sqrt[3]{(y+1)^2+27}\,.$$

Note that the domain of $f(x)$ *is* $x \geq 3$, *and its range is* $f(x) \in [-1,\infty)$. *Therefore the domain of* $f^{-1}(y)$ *is* $y \in [-1,\infty)$. *We have*

$$f^{-1}(x) = \sqrt[3]{(x+1)^2+27}, \quad x \geq -1\,.$$

3. *We solve*

$$y = \sqrt{x^2-2x+1} = \sqrt{(x-1)^2} = |x-1|\,.$$

And this function has no inverse, since eqch value of $y > 0$ *has two possible solutions.*