MA1E01: Solutions week 2

Solution 1

1. *The function can take any real number as input except $x = 43$ (would produce a division by zero), or any number smaller than 8 (would produce the square root of a negative number). So the domain of $f(x)$ is given by the values of x such that*

$$x \in (8, \infty)$$

2. *The argument of the square root is $(t+7)^2 \geq 0$, so this can never be negative. The denominator can vanish at $t = 1$ or $t = 3$. So the domain of $f(t)$ is given by the values of t such that*

$$t \in \mathbb{R} \setminus \{1, 3\}\,.$$

or

$$t \in (-\infty, 1) \cup (1, 3) \cup (3, \infty)\,.$$

Solution 2

1. *The numerator is just a (scary) number. Therefore the function can take any real number as input except $x = 3$ (would produce a division by zero), or any number larger than 3 (would produce the square root of a negative number). So the domain of $f(x)$ is given by the values of x such that*

$$x \in (-\infty, 3)$$

2. *The argument of the square root is $h^2 - 2h + 1 = (h-1)^2 \geq 0$, so this can never be negative. The denominator $h - \sqrt{\pi}$ vanish at $h = \sqrt{\pi}$. So the domain of $g(h)$ is given by the values of h such that*

$$t \in \mathbb{R} \setminus \{\sqrt{\pi}\}\,.$$

or

$$t \in (-\infty, \sqrt{\pi}) \cup (\sqrt{\pi}, \infty)\,.$$

3. *For all three functions $s(x), m(x), p(x)$, the domain is just the intersection of the domains of $f(x)$ and $g(x)$, given by*

$$x \in (-\infty, \sqrt{\pi}) \cup (\sqrt{\pi}, 3)\,.$$

In the case of $r(x)$, we have also to exclude $x = 1$, since $g(1) = 0$, so the domain is

$$x \in (-\infty, 1) \cup (1, \sqrt{\pi}) \cup (\sqrt{\pi}, 3)\,.$$

In the case of $d(x)$ the domain is

$$x \in (-\infty, \sqrt{\pi}) \cup (\sqrt{\pi}, 3)\,.$$

since $f(x)$ can never be zero.

Solution 3 *The domain of $f(x)$ is $x \geq 3$, while the domain of $g(x)$ is all the real numbers. Therefore for the first three cases, we have*

- $f(x) + g(x) = \sqrt{x-3} + x^2 + 4$
- $f(x) - g(x) = \sqrt{x-3} - x^2 - 4$
- $f(x)g(x) = (x^2+4)\sqrt{x-3}$

and the domain is just the intersection of the domains

$$x \geq 3\,.$$

For the case

$$\frac{g(x)}{f(x)} = \frac{x^2+4}{\sqrt{x-3}}$$

We have to exclude also $x = 3$, since $f(3) = 0$. So the domain of $g(x)/f(x)$ is

$$x > 3\,.$$

Finally we have

$$(f \circ g)(x) = f(g(x)) = f(x^2+4) = \sqrt{x^2+1}\,, \tag{1}$$

and the domain are all the real numbers. On the other hand

$$(g \circ f)(x) = g(f(x)) = g(\sqrt{x-3}) = x+1 \qquad (x \geq 3)\,, \tag{2}$$

and the domain is

$$x \geq 3\,.$$

Solution 4 *The domain of $f(x)$ is $x \neq 1$, while the domain of $g(x)$ is all the real numbers. Therefore for the first three cases, we have*

- $f(x) + g(x) = \frac{2}{x-1} + x^2 + 2$
- $f(x) - g(x) = \frac{2}{x-1} - x^2 + 2$
- $f(x)g(x) = \frac{2x^2+2}{x-1}$
- $\frac{g(x)}{f(x)} = \frac{1}{2}(x^3 - x^2 + 2x - 2)\,, \quad (x \neq 1)$

and the domain is just the intersection of the domains

$$x \neq 1\,.$$

Finally we have

$$(f \circ g)(x) = f(g(x)) = f(x^2+2) = \frac{2}{x^2+1}\,, \tag{3}$$

and the domain are all the real numbers. On the other hand

$$(g \circ f)(x) = g(f(x)) = g\left(\frac{2}{x-1}\right) = \frac{4}{(x-1)^2} + 2 \qquad (x \neq 1)\,, \tag{4}$$

and the domain is

$$x \neq 1\,.$$