

Recap: Limits

- We write

$$\lim_{x \rightarrow a} f(x) = L$$

if **f approaches** L close to a .

- The limit of f at a has **nothing** to do with the value of the function at a . The limit can exist even if the function is not defined. For example:

$$\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1} = 2,$$

but the function $\frac{x^2 - 1}{x - 1}$ is not even defined at $x = 1$. (See example 3, section 2.1).

- Right and left limits mean (see example 2, section 2.1):

$$\lim_{x \rightarrow a^+} f(x) = L \implies f \text{ approaches } L \text{ close to } a \text{ for values larger than } a.$$

$$\lim_{x \rightarrow a^-} f(x) = L \implies f \text{ approaches } L \text{ close to } a \text{ for values smaller than } a.$$

- A limit exists **if and only if** the right and left limits exist **and** are the same.

$$\lim_{x \rightarrow a} f(x) = L \iff \begin{cases} \lim_{x \rightarrow a^+} f(x) = L \\ \lim_{x \rightarrow a^-} f(x) = L \end{cases}$$

- If a function is continuous at a , then the limit at a is given by the value of the function (Examples 4, 5 of section 2.1).

$$f(x) \text{ continuous at } a \implies \lim_{x \rightarrow a} f(x) = f(a).$$

- Polynomials and the trigonometric functions $\sin x, \cos x$ are continuous at every point. The function \sqrt{x} is continuous at $x > 0$. The sum, difference and composition of continuous functions is continuous. The ratio of continuous functions $f(x)/g(x)$ is continuous if $g(x) \neq 0$. (Examples 1, 2, 3 of section 2.2).
- Computing limits.

1. If the function is continuous, just evaluate the function. Example:

$$\lim_{x \rightarrow 2} \sqrt{x^3 + 12x + 4} = 6.$$

- If the function is **not** continuous, is probably of the form $\lim_{x \rightarrow a} f(x)/g(x)$ with $\lim_{x \rightarrow a} g(x) = 0$. In this case

– If $\lim_{x \rightarrow a} f(x) \neq 0$ the limit does not exist (Example 7, section 2.2).

– If $\lim_{x \rightarrow a} f(x) = 0$ one needs to simplify the expression, assuming that $x \neq a$ (Example 4, section 2.2).

- $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$