2018-10-04 Thu
Chapter

Sequences

- Sequences are functions that only accept integer values as input. They

can be seen as a list of numbers $a_1, a_2, a_3, \ldots$. The function that characterizes this sequence is $f(n) = a_n$

- Example: the first few terms of the sequence

$$a_n = \frac{n}{n+1}\,, \qquad (n \geq 0)\,. \tag{1}$$

are $0, 1/2, 2/3, 3/4, \ldots$.

- The graph consist of points. No continuity for sequences, only limits at $\pm\infty$ make sense for sequences!

Recursive definition of sequences

- Simple example

$$a_{n+1} = \frac{1}{2}(a_n + 6)\,. \qquad a_1 = 2\,. \tag{2}$$

We have the sequence

$$2, 4, 5, 5.5, 5.75, \ldots \tag{3}$$

- Famous example: Fibonacci sequence

$$F_n = F_{n-1} + F_{n-2}\,. \qquad F_1 = F_2 = 1\,. \tag{4}$$

Gives the numbers

$$1, 1, 2, 3, 5, 8, 13, 21, \ldots \tag{5}$$

Arithmetic and geometric sequences

Arithmetic series

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \tag{6}$$

Geometric series

$$\sum_{i=0}^{n} r^i = \frac{1 - r^{n+1}}{1 - r} \tag{7}$$