2018-10-01 Mon
Chapter 1.5

One sided limits

- We say that

$$\lim_{x \to a^+} f(x) = L \tag{1}$$

if f approaches L close to a (but only with values greater than a).

- We say that

$$\lim_{x \to a^-} f(x) = L \tag{2}$$

if f approaches L close to a (but only with values smaller than a).

Examples

- A simple example

$$\lim_{x \to 0^+} \sqrt{x} = 0 \tag{3}$$

but $\lim_{x \to 0} \sqrt{x}$ does not exist.

- Pice-wise defined functions

$$f(x) = \begin{cases} \frac{(x-1)^2}{x^2-1} & x > 1 \\ x^2 & x < 1 \end{cases} \tag{4}$$

- Another example

$$f(x) = \begin{cases} \frac{x-1}{x^2-2x+1} & x > 1 \\ x^2 & x < 1 \end{cases} \tag{5}$$

- Compute the limit

$$\lim_{x \to 0} \frac{|x|}{x} \tag{6}$$

Continuity

We say that $f(x)$ is continuous at a if

$$\lim_{x \to a} f(x) = f(a) \tag{7}$$

Properties

Continuity preserved by usual operations as long as you do not divide by zero.

A very important theorem

If f is continuous in $[a, b]$ and $f(a)f(b) < 0$ then there exist a $c \in (a, b)$ such that $f(c) = 0$.