2018-09-27 Thu
Chapter 1.4

A special limit

This limit has to be remembered

$$\lim_{x\to 0} \frac{\sin x}{x} = \lim_{x\to 0} \frac{x}{\sin x} = 1\,. \tag{1}$$

The sandwich theorem

If $f(x) \leq h(x) \leq g(x)$ and $\lim_{x\to a} f(x) = \lim_{x\to a} g(x) = L$ then

$$\lim_{x\to a} h(x) = L\,. \tag{2}$$

Basic rules

These following rules apply to the computation of limits

$$\lim_{x\to a}[f(x) \pm g(x)] = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x)\,. \tag{3}$$

$$\lim_{x\to a}[f(x)g(x)] = [\lim_{x\to a} f(x)][\lim_{x\to a} g(x)]\,. \tag{4}$$

$$\lim_{x\to a} g(x) \neq 0 \Longrightarrow \lim_{x\to a}\left[\frac{f(x)}{g(x)}\right] = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}\,. \tag{5}$$

$$\lim_{x\to a} f^n(x) = [\lim_{x\to a} f(x)]^n\,. \tag{6}$$

$$\lim_{x\to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to a} f(x)}\,. \tag{7}$$

In the last case we require that $\lim_{x\to a} f(x) > 0$ when n is even