2018-09-24 Mon
Chapter 1.4

Definition

We say that

$$\lim_{x \to a} f(x) = L \tag{1}$$

if for all $\epsilon > 0$, there exist a $\delta > 0$ such that

$$0 < |x - a| < \delta \implies |f(x) - L| < \epsilon. \tag{2}$$

Some examples

1. We say that $\lim_{x \to 1} 2x + 1 = 3$, because for every $\epsilon > 0$, choosing $\delta = \epsilon/2$, for $0 < |x - 1| < \delta = \epsilon/2$ we have

$$|f(x) - L| = |2x + 1 - 3| = 2|x - 1| < 2\epsilon/2 < \epsilon \tag{3}$$

1. We say that $\lim_{x \to 1} \frac{x^2 - 1}{x - 1} = 2$, because for every $\epsilon > 0$, choosing $\delta = \epsilon$, for $0 < |x - 1| < \delta$ we have

$$|f(x) - L| = \left| \frac{x^2 - 1}{x - 1} - 2 \right| = |x - 1| < \epsilon \tag{4}$$