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Chapter 1.4/1.5

Speed of a falling object

- Galileo realized that the distance travelled by a falling object is very well approximated by the function

$$d(t) = 5t^2 \tag{1}$$

- What is the average speed of an object around $t = 1$? Distance traveled over time

$$v_{\text{av}}(t) = \frac{d(t) - d(1)}{t - 1} = \frac{5t^2 - 5}{t - 1} \tag{2}$$

$d(1)$	t	$d(t)$	$v_{\text{av}}(t)$
5	1.2	7.2	11.
5	1.1	6.05	10.5
5	1.01	5.1005	10.05
5	1.001	5.010005	10.005
5	0.999	4.990005	9.995
5	0.99	4.9005	9.95
5	0.9	4.05	9.5
5	0.8	3.2	9.
5	0.7	2.45	8.5

- One might be tempted to just cancel the zero in the numerator/denominator, but this cannot be done in general!

$$f(x) = \frac{5x^2 - 5}{(x - 1)^2} \,. \tag{3}$$

x	$f(x)$
1.2	55.
1.1	105.
1.01	1005.
1.001	10005.
0.999	-9995.
0.99	-995.
0.9	-95.
0.8	-45.

- Similarities: Both functions $f(x)$ and $v_{\text{av}}(x)$ are not defined at $x = 1$ (i.e. $x = 1$ is not in the domain).

- Difference between two situations: v_{av} **aproaches** 10 close to 1, but f does not approach any reasonable number close to 1.

- Definition of limit: We say that

$$\lim_{x \to a} f(x) = L \tag{4}$$

if f approaches L close to a.

- Computing the limit: Since we have

$$v_{\rm av}(t) = \frac{5t^2-5}{t-1} = \begin{cases} 5(t+1) & t \neq 1 \\ \text{Not defined} & t = 1 \end{cases} \tag{5}$$

Obviously $v_{\rm av}$ approaches 10 close to 1!:

$$\lim_{t\to 1} \frac{5t^2-5}{t-1} = \lim_{t\to 1} \frac{5(t+1)(t-1)}{t-1} = \lim_{t\to 1} 5(t+1) = 10\,. \tag{6}$$

On the other hand

$$f(x) = \frac{5x^2-5}{(x-1)^2} = \frac{5(x+1)}{(x-1)}\,. \tag{7}$$

and close to $x=1$ the function grows/decrease without bound (i.e. does not approach any number). We write

$$\lim_{x\to 1} f(x) = \text{Does not exist}\,. \tag{8}$$