

Transformation of functions as compositions

Translations

Define the function $g(x) = x + c$, with c some constant. Now for any function $f(x)$, the composition

$$h = (f \circ g) \tag{1}$$

Is a function just like $f(x)$ but shifted an amount $-c$ in the horizontal direction.

The function

$$h = (g \circ f) \tag{2}$$

Is a function just like $f(x)$ but shifted an amount c in the vertical direction.

Stretching

Define the function $g(x) = cx$, with c some constant. Now for any function $f(x)$, the composition

$$h = (f \circ g) \tag{3}$$

Is a function just like $f(x)$ but stretched by a factor $1/c$ in the horizontal direction.

The function

$$h = (g \circ f) \tag{4}$$

Is a function just like $f(x)$ but stretched a factor c in the vertical direction.