1 One sided limits and countinuity (W04/1)

1.1 One sided limits

- We say that

$$\lim_{x \to a^+} f(x) = L \tag{1}$$

if f approaches L close to a (but only with values greater than a).

- We say that

$$\lim_{x \to a^-} f(x) = L \tag{2}$$

if f approaches L close to a (but only with values smaller than a).

1.1.1 Examples

- A simple example

$$\lim_{x \to 0^+} \sqrt{x} = 0 \tag{3}$$

but $\lim_{x \to 0} \sqrt{x}$ does not exist.

- Pice-wise defined functions

$$f(x) = \begin{cases} \frac{(x-1)^2}{x^2-1} & x > 1 \\ x^2 & x < 1 \end{cases} \tag{4}$$

- Another example

$$f(x) = \begin{cases} \frac{x-1}{x^2-2x+1} & x > 1 \\ x^2 & x < 1 \end{cases} \tag{5}$$

- Compute the limit

$$\lim_{x \to 0} \frac{|x|}{x} \tag{6}$$

1.2 Continuity

We say that $f(x)$ is continuous at a if

$$\lim_{x \to a} f(x) = f(a) \tag{7}$$

1.2.1 Properties

Continuity preserved by usual operations as long as you do not divide by zero.

1.2.2 A very important theorem

If f is continuous in $[a, b]$ and $f(a)f(b) < 0$ then there exist a $c \in (a, b)$ such that $f(c) = 0$.

2 Limits at infinity (W04/2)

2.1 Basics

- We say that

$$\lim_{x\to\infty} f(x) = L \tag{8}$$

If f approaches L at arbitrarily large values of the argument.

- Remember

$$\lim_{x\to\pm\infty} \frac{1}{x^p} = 0; \qquad (p > 0)\,. \tag{9}$$

and

$$\lim_{x\to\pm\infty} x^p = \text{Does nto exists}; \qquad (p > 0)\,. \tag{10}$$

Also none of the trigonometric limits

$$\lim_{x\to\pm\infty} \sin(x), \lim_{x\to\pm\infty} \cos(x), \lim_{x\to\pm\infty} \tan(x) \tag{11}$$

exist.

2.2 Computing limits at ∞

When dealing with rational functions, divide by the higer power

$$\lim_{x\to\pm\infty} \frac{x^2-3}{2x^2+4x-1} = \lim_{x\to\pm\infty} \frac{x^2/x^2-3/x^2}{2x^2/^2+4x/x^2-1/x^2} = \lim_{x\to\pm\infty} \frac{1-3/x^2}{2+4/x-1/x^2} = \frac{1}{2} \tag{12}$$

2.3 Compound interest

1% rate per year with n compounding frequency

n	(1. + 1. / n)^{n}
1	2.
2	2.25
6	2.5216264
12	2.6130353
24	2.6637313
365	2.7145675
31556952.	2.7183900

3 Sequences (W04/3)

3.1 Sequences

- Sequences are functions that only accept integer values as input. They

can be seen as a list of numbers $a_1, a_2, a_3, \ldots$. The function that characterizes this sequence is $f(n) = a_n$

- Example: the first few terms of the sequence

$$a_n = \frac{n}{n+1}\,, \qquad (n \geq 0)\,. \tag{13}$$

are $0, 1/2, 2/3, 3/4, \ldots$.

- The graph consist of points. No continuity for sequences, only limits at $\pm\infty$ make sense for sequences!

3.1.1 Recursive definition of sequences

- Simple example

$$a_{n+1} = \frac{1}{2}(a_n + 6)\,. \qquad a_1 = 2\,. \tag{14}$$

We have the sequence

$$2, 4, 5, 5.5, 5.75, \ldots \tag{15}$$

- Famous example: Fibonacci sequence

$$F_n = F_{n-1} + F_{n-2}\,. \qquad F_1 = F_2 = 1\,. \tag{16}$$

Gives the numbers

$$1, 1, 2, 3, 5, 8, 13, 21, \ldots \tag{17}$$

3.2 Arithmetic and geometric sequences

Arithmetic series

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \tag{18}$$

Geometric series

$$\sum_{i=0}^{n} r^i = \frac{1 - r^{n+1}}{1 - r} \tag{19}$$