1 Elementary functions (I) (W02/1)

Linear functions

$$f(x) = mx + n \tag{1}$$

- m : slope. $m > 0$ "increasing" line. $m < 0$ "decreasing" line
- n : intercept. (Cut with the vertical axes)

Quadratic functions

$$f(x) = ax^2 + bx + c \tag{2}$$

Are parabolas

- $a > 0$ opens up
- $a < 0$ opens down
- Minima at $\frac{-b}{2a}$.

Polynomials

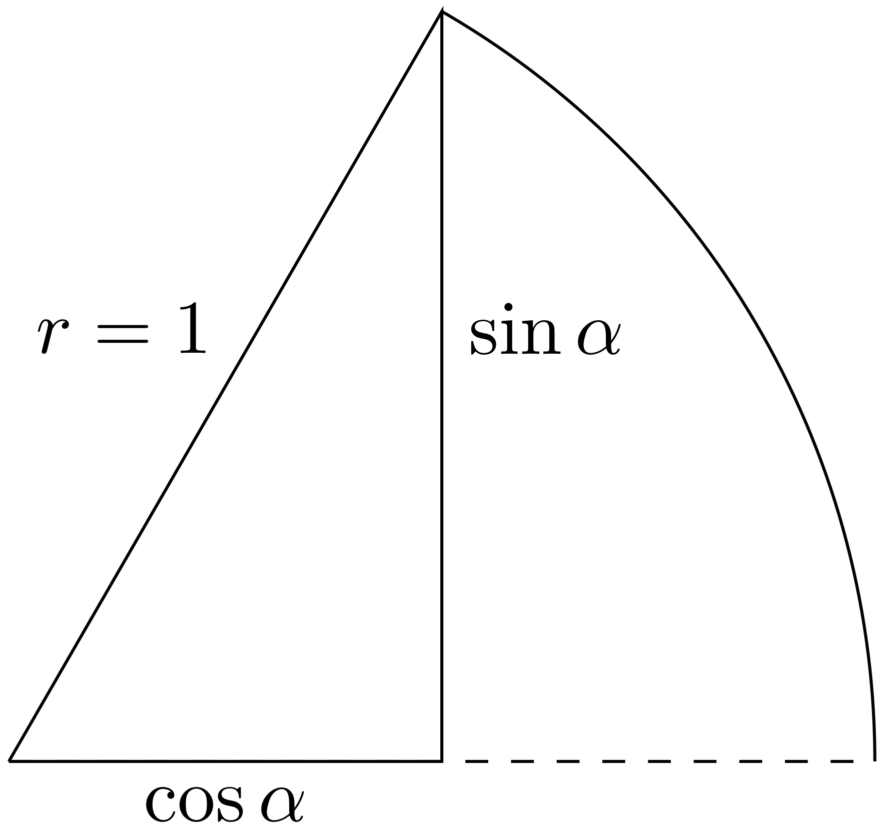
$$f(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n \tag{3}$$

- n : degree of polynomial
- Always go to $\pm\infty$ as $x \to \infty$ or $x \to -\infty$. Depending on the sign of a_n and if the degree is even/odd.

Trigonometric

- Useful to describe periodic phenomena. Different from polynomials, that eventually go to $\pm\infty$
- Angles measured in radians:

$$\text{angle} = \frac{\text{arc length of circumference}}{\text{radius of circumference}} \tag{4}$$



- $\sin(x)$: In a circle of radius one, is the length of the opposite side.
- $\cos(x)$: In a circle of radius one, is the length of the adjacent side.
- By Pythagoras theorem $\sin^2 \alpha + \cos^2 \alpha = 1$
- Some special values

α	$\sin \alpha$	$\cos \alpha$
$\pi/2$	1	0
π	0	-1
$3\pi/2$	-1	0

Elementary functions (II) (W02/2)

"Anyone who believes exponential growth can go on forever in a finite world is either a madman or an economist." –Kenneth Boulding–

Exponential growth

Birth rate ($r = 0.1$), mortality rate ($m = 0.07$). Population after n years

$$P_n = 1.03^n P_0 \tag{5}$$

n	$P_n/P_{\max}$	Left
0	1 (-6)	0.999999
10	1.3439164 (-6)	0.99999866
100	1.9218632 (-5)	0.99998078
200	3.6935582 (-4)	0.99963064
300	7.0985135 (-3)	0.99290149
400	0.13642372	0.86357628
430	0.33113617	0.66886383
450	0.59806876	0.40193124
460	0.80375440	0.1962456
465	0.93177164	0.06822836
467	0.98851653	0.01148347
468	1.0181720	-0.018172

Exponential functions grow very fast!!!!

Exponential functions

$$f(x) = 2^x \tag{6}$$

- Clear what is meant when

$$x$$

  is integer

- Also clear when x is a rational number
- But what is 2^π, or $3^{\sqrt{2}}$?
- Definition of exponential functions is complicated. Rigorous definition requires integrals!!!!

Rules with exponents

$$a^x a^y = a^{x+y}; \quad (a^x)^y = a^{xy} \tag{7}$$

Logarithmic functions

$$y = \log_b x \Longleftrightarrow b^y = x \tag{8}$$

Defined as the inverse function of the exponential

Rules with logarithms

$$\log(xy) = \log x + \log y; \qquad \log(x^y) = y \log x\,. \tag{9}$$

Elementary functions (III) (W02/3)

Inverse functions: f^{-1} "undoes" the operations done by f. Example:

$$f(x) = x^3 + 1 \Longrightarrow f^{-1}(x) = \sqrt[3]{y-1} \tag{10}$$

Important: Some functions do not have an inverse. Example:

$$f(x) = x^2\,. \tag{11}$$

Since both $f(a) = f(-a)$ (for all $a \neq 0$) We can never undo the work done by f: How to choose between positive and negative option?