"Calculus". Anton, Bivens, Davies: Chapter 0.2
"Biocalculus". Steward, Day: Chapter 1.3

1 Introduction, functions (W01/1)

Practicalities

- All information on the subject in https://www.maths.tcd.ie/~alberto.ramos/teaching/ma1e01_18/ma1e01/

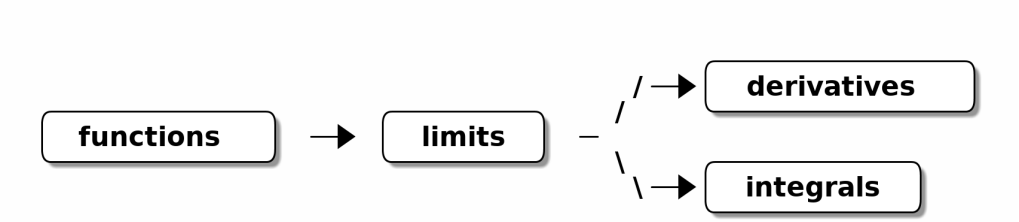
Evaluation

1. 80% Written exam.
2. 20% Continuous Assessment.

Calculus

Historically

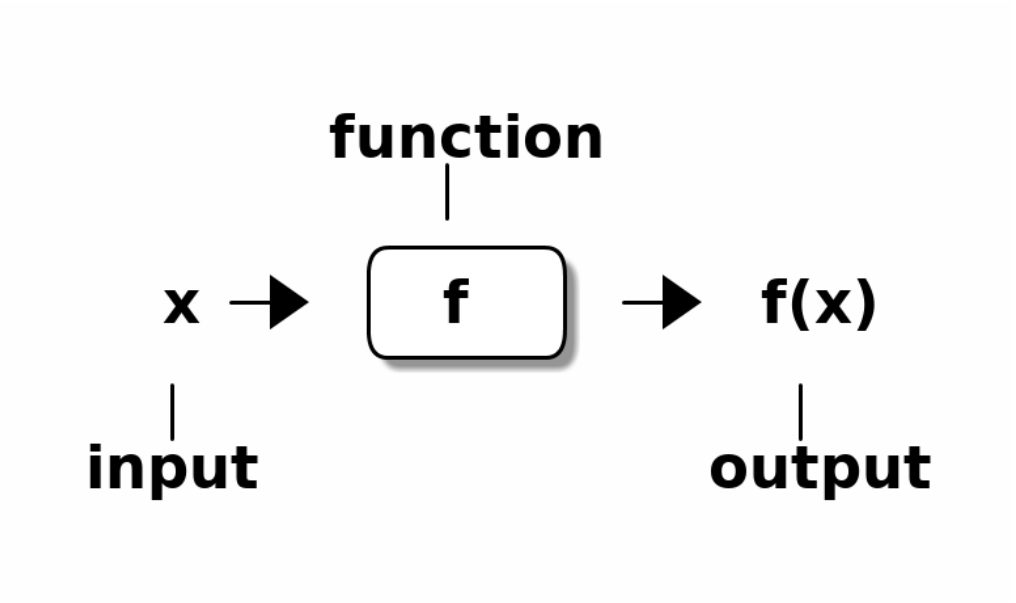
- Need to describe **motion**. How is motion possible?
- Calculus: provides methods for quantitative investigation of change.
- Main objective of the course: derivation/integration. **But** to understand this we need to master functions/limits



Functions

Intuitive idea

- How one quantity depends on another
  - Distance traveled by a free falling object is (established experimentally by Galileo!)
$$d(t) = \kappa t^2 \tag{1}$$
  - Position of earth orbiting around the Sun along the year
  - Volume of a box constructed from a rectangle of side L
$$V(x) = x(L-2x)^2 \tag{2}$$
  - Graph of a function
  - Table of values (never exhaustive)
- Always think of functions as "machines". Three basic ingredients: input, the function, output.



Questions in calculus

- What is the velocity of an object after 2sec falling?
- How much distance travels the Earth in one year?
- How do I choose x to maximize the volume of the Box?

Domain of a function

Acceptable values for the input

- Not all numerical values acceptable for the independent variable. (What is the volume of the box for $x = -1$?)
- Possible values restricted for two main reasons
  1. The function cannot be computed (natural domain)
  $$f(x) = \sqrt{1 - x^2}; \qquad f(x) = \frac{1}{1 - x}\,. \tag{3}$$
  at $x = 1$.
  2. Even if the value is possible **in principle**, the nature of the problem restricts some values. (i.e. x is a distance, and therefore always positive).
- Domain of a function: always be careful with operations that cannot be done
  - Square roots of negative numbers
  - Fractions with zero denominator
  - log of a negative numbers

Domain, new functions from old functions (W01/2)

Functions

- Graphs, Tables, equations
- Not every graph is a function (vertical line test)
- picewise defined functions

Domain and intervals

- Remember: set of allowed values for a function.
- Be careful with
  - Square roots of negative numbers
  - Fractions with zero denominator
  - ...
- Intervals:
  - $[,]$
  - $(,)$
  - $(,]$
  - $[,)$

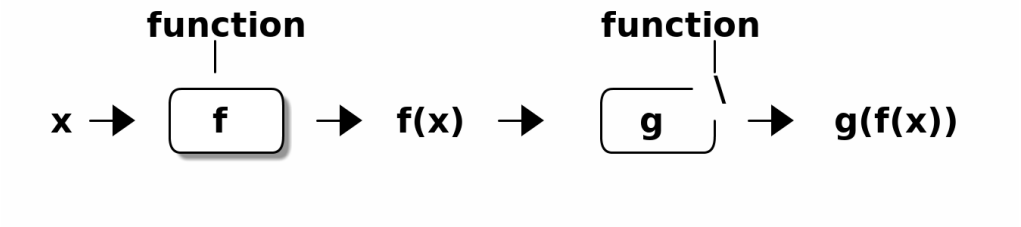
New functions from old functions

Basic arithmetic operations

- $f(x) \pm g(x)$: Domain intersection of domains
- $f(x) * g(x)$: Domain intersection of domains
- $f(x)/g(x)$: Domain intersection of domains with values that make $g = 0$ excluded

Compsition of functions

- $(g \circ f)(x) = g(f(x))$



Transformation of functions (W01/3)

Domain under composition

Consider the exmaple

$$f(x) = x^2 + 1; \qquad g(x) = \sqrt{x}\,. \tag{4}$$

Now the function

$$h = (f \circ g) \tag{5}$$

is given by

$$h(x) = f(g(x)) = f(\sqrt{x}) = (\sqrt{x})^2 + 1 = x + 1\,. \tag{6}$$

Bot one has to be very carefull: The function h is defined by the composition so x has to be in the domain of g. It is more correct to say

$$h(x) = x + 1\,, \qquad \text{for } x \leq 0\,. \tag{7}$$

On the other hand

$$u = (g \circ f) = g(f(x)) = g(x^2 + 1) = \sqrt{x^2 + 1} \tag{8}$$

And domain is all numbers.

A final example:

$$f(x) = -x; \qquad g(x) = \sqrt{x}\,. \tag{9}$$

with

$$(g \circ f)(x) = \sqrt{-x}\,, \qquad (f \circ g)(x) = -\sqrt{x} \tag{10}$$

have different domains!

Transformation of functions as compositions

Translations

Define the function $g(x) = x+c$, with c some constant. Now for any function $f(x)$, the composition

$$h = (f \circ g) \tag{11}$$

Is a function just like $f(x)$ but shifted an amount $-c$ in the horizontal direction.

The function

$$h = (g \circ f) \tag{12}$$

Is a function just like $f(x)$ bu shifted an amount c in the vertical direction direction.

Stretching

Define the function $g(x) = cx$, with c some constant. Now for any function $f(x)$, the composition

$$h = (f \circ g) \tag{13}$$

Is a function just like $f(x)$ but stretched by a factor $1/c$ in the horizontal direction.

The function

$$h = (g \circ f) \tag{14}$$

Is a function just like $f(x)$ but stretched a factor c in the vertical direction direction.