Module Code	MAU23101
Module Name	Introduction to Number Theory
ECTS Weighting	5 ECTS
Semester taught	Semester 1
Module Coordinator/s	Prof Nicolas Mascot
Module Learning Outcomes with embedded Graduate Attributes	On successful completion of this module, students should be able to: LO1. State and prove theorems in number theory. LO2. Use these theorems to solve number theoretic problems, such as some classes of Diophantine equations.
Module Content	Number theory is the study of the property of the integers, mainly in view of attempting to solve Diophantine equations, that is to say equations whose unknowns are required to assume integer values. In general, such equations are extremely difficult to solve; the goal of this course is to introduce some basic techniques needed to tackle simple cases. For instance, by the end of this course, students will be able to show that the equation $x^3 + y^3 + z^3 = 31$ has no solutions in integers, and to explain why 2017 can be expressed as a sum of two squares, whereas 2016 can be expressed as a sum of three squares but not as a sum of two, and 2015 can be expressed as a sum of four squares but not three. The topics to be covered are: 1. Divisibility and factorisation of the integers Prime numbers, gcd and lcm, Euclid's algorithm, Bézout's theorem, multiplicative functions such as sums of divisors. 2. Congruences Arithmetic in the ring Z/nZ and in the field Z/pZ, Euler's totient function

	ϕ(n), Chinese remainders, multiplicative order and primitive roots. 3. Power residues mod p Legendre symbol, quadratic reciprocity, quadratic equations mod p. 4. Sums of squares Integers that are the sum of 2 or 3 squares, every integer is the sum of 4 squares. 5. Quadratic forms Equivalence of quadratic forms, discriminant, integers represented by quadratic forms, class numbers. 6. Continued fractions Continued fraction expansion of rationals and of quadratic irrationals, Diophantine approximation, Pell-Fermat equations.
Teaching and Learning Methods	The lectures include numerous example so as to illustrate each concept. A vast sample of practice exercises is available so as to offer the possibility of improving problem solving skills. Students are encouraged to use LaTteXx for submitted work.

Assessment Details

Assessment Component	Assessment Description	LO Addressed	% of total	Week due
Exam	2 hour written exam	LO 1-3	85	
Continuous	Homeworks	LO 1-3	15	

Reassessment Requirements

Reassessment is done by means of a two hour exam in the supplemental session.

Contact Hours and Indicative Student Workload

Contact hours: 11x3hours lectures
Independent Study (preparation for course and review of materials): 46 hours
Independent Study (preparation for assessment, incl. completion

	of assessment): 46 hours
Recommended Reading List	• *A course in computational number theory*, by D. Bressoud and S. Wagon • *The higher arithmetic*, by H. Davenport (up to chapter VI) • *A classical introduction to modern number theory*, by K. Ireland and M. Rosen (up to chapter 5) • *Primes of the form x^2+ny^2*, by D. Cox (up to section 2.B)
Module Pre-requisite	MAU11102
Module Co-requisite	None
Module Website	https://www.maths.tcd.ie/~mascotn/teaching/2020/MAU23101/
Are other Schools/Departments involved in the delivery of this module? If yes, please provide details.	No