School of Mathematics

Topics in Algebra - Graphs and Groups. 1999-2000
(JS and SS Mathematics.)

Lecturer: Dr. M. Batty.

Requirements/prerequisites: Elementary group theory.

Duration: 21 Weeks.

Number of lectures per week: 3

Assessment: There will be some assignments.

End-of-year Examination: 3-hour end of year exam.

Description: The fundamental objects of study in this course will be *presentations* of groups. This is how groups often arise in other branches of mathematics, yet it is usually the case that no general algorithm exists to answer particular questions about a presentation, e.g. is the corresponding group trivial or not?
A modern approach is to consider groups as geometric objects, rather than purely algebraic, via a device known as a *Cayley graph.* This outlook makes it possible to understand and exploit connections between group theory, topology, geometry and even computer science.
I plan to spend the first half of the course on the basics of free groups, group presentations, graphs, groups acting on graphs, free products with amalgamation and HNN Extensions, and connections between group presentations and topology. The second half is going to deal with more advanced topics, which may include some of the following: Bass-Serre theory (groups acting on trees), Stallings' ends theorem, Dunwoody's accessibility theorem, the Muller-Schupp theorem on groups with a context-free word problem language.

Textbooks: No book will cover the subject matter for the entire course. A good introduction to presentations and free groups is *A Course in the Theory of Groups* by Derek Robinson (Springer GTM no. 80). Only chapters 2 and 6 (out of 15!) are really relevant. There are many books on graphs. One I particularly like is the book by F. Harary, called "Graph Theory".

October 5, 1999