**School of Mathematics**

**Course 311 — Abstract Algebra** 1999-2000
(Optional JS & SS Mathematics, SS Two-subject Moderatorship )

**Lecturer:** Dr. D.R. Wilkins

**Requirements/prerequisites:** 111

**Duration:** 21 weeks

**Number of lectures per week:** 3

**Assessment:**

**End-of-year Examination:** One 3-hour examination

**Description:**

This course continues the study of groups, rings and fields commenced in course 111. A large part of the course is devoted to *Galois theory*, in which techniques of modern algebra are applied to the problem of expressing the roots of a polynomial as functions of its coefficients. To any polynomial is associated a finite group, referred to as the *Galois group* of the polynomial. The roots of a polynomial can be expressed in terms of its coefficients by means of algebraic formulae involving only the operations of addition, subtraction, multiplication, division and the extraction of $n$th roots if and only if the Galois group of the polynomial is 'solvable'. This result can be used to prove that there cannot exist any algebraic formula for the roots of a general quintic polynomial that involves only the algebraic operations of addition, subtraction, multiplication, division and the extraction of $n$th roots.

The course will also study certain topics in number theory, including the Chinese Remainder Theorem and the Law of Quadratic Reciprocity.

1. *Groups*: basis properties of groups, permutation groups, the Sylow theorems, solvable groups, the classification of finitely-generated Abelian groups.

2. *Number theory*: congruences, the quadratic reciprocity law.

3. *Galois theory*: Factorization of polynomials, field extensions, splitting fields, Galois groups of field extensions and of polynomials, solvability of polynomial equations.

**Books:**

John B.Fraleigh, *A first course in abstract algebra.*
Ian Stewart, *Galois theory.*
H.M. Edwards, *Galois theory.*

May 6, 1999