School of Mathematics

Course 442 - General Relativity 1998-99
(Optional JS & SS Theoretical Physics, JS & SS Mathematics)

Lecturer: P.S. Florides

Requirements/prerequisites: 211, 221, 241

Duration: 21 weeks.

Number of lectures per week: 3

Assessment:

End-of-year Examination: One 3-hour examination

Description: *Mathematical Background*
Linear and Multilinear algebra, general tensors, tensor products, tests for tensor character, inner product, associated tensors (overview). Differentiable manifolds $(M_1, M', \ldots)$, differentiable functions, differentiable mappings $\phi\colon M \to M'$, tangent vectors, tangent spaces and their duals (briefly).

The differential (ϕ_*) of a map $\phi\colon M \to M'$ and its dual (ϕ^*). Vector fields and their commutator product, 1-form fields, tensor fields, moving frames. Differentiable curves, and vector fields on such curves, integral curves, r-surfaces.

Affine manifolds, connection coefficients, covariant derivatives of vector, 1-form and tensor fields, parallel propagation along a differentiable curve, intrinsic derivatives , Geodesics. The Riemann tensor and the torsion tensor.

Riemannian manifolds, the Riemann connection, properties of the Riemann tensor; the Ricci and Einstein tensors. The necessary and sufficient condition for a Riemannian manifold to be flat.

Equation of geodesic deviation, stationary properties of geodesics, Riemannian curvature, spaces of constant curvature, Schurs theorem.

General Relativity (G.R.)
Physical foundations of G.R., space-time as a Riemannian manifold, Einstein's field equations. The linearized Einstein field equations, Newton's theory as a first approximation, further (exact) analogies between Newton's and Einstein's theories.
(Digression: Lie derivatives, groups of motions, killing vectors. General form of the metric with spherical symmetry).

Exact solutions of Einstein's equations. The Schwarzschild exterior solution, Birkhoff's theorem, the three "crucial tests", the generalized red-shift formula. Study of the Schwarzschild radius, the Eddington - Finkelstein coordinates, Kruskal coordinates, black holes.

Dynamics of perfect fluids, the Oppenheimer-Volkov equation, maximum mass, the interior Schwarzschild solution; the Einstein and de Sitter Universes.

Electromagnetic Theory
The electromagnetic tensor, the electromagnetic energy tensor, the combined Einstein-Maxwell field equations, motion of charged particles in a electromagnetic field (in curved space-time), the Reissner-Nordström solution.

[Selected topics (time permitting): variational principles and conservation laws, (linearized) gravitational radiation, standard cosmology].

June 10, 1998