School of Mathematics

Course 321 — Modern Analysis 1998-99
(Optional JS & SS Mathematics, SS Two-subject Moderatorship)

Lecturer: Dr. R.M. Timoney

Requirements/prerequisites: 212

Duration: 21 weeks

Number of lectures per week: 3

Assessment: Regular assignments.

End-of-year Examination: One 3-hour examination

Description: *Fundamental Concepts*: Partial order, Zorn's lemma as an axiom, application to bases of vector spaces; cardinal numbers; ordinal numbers.
General Topology: Neighbourhoods, first countable, inadequacy of sequences, second- countable, (relationship to separability), continuity of functions at points, product topology (weak topology for continuous projections).

Nets, advantages over sequences, subnets; Hausdorff separation axiom, Urysohn's lemma, Tietze extension. Compactness via nets, Tychonoff's theorem (compactness of products), compactification (Stone-Čech and universal properties, one-point), local compactness, completions of metric spaces, Baire category theorem.
Functional Analysis:

Banach spaces: definitions and examples ($C_0(X)$, ℓ_∞, $C(K)$, Hölder and Minkowski inequalities, ℓ_p, closed subspaces,c_0, $L_p(\mathbb{R})$, $L_p[0,1]$).

Linear operators: examples of continuous inclusions among ℓ_p and $L_p[0,1]$ spaces, n-dimensional normed spaces isomorphic. Open mapping and closed graph theorems. Uniform boundedness principle.

Dual spaces: Hahn-Banach theorem, canonical isometric embedding in double dual, reflexivity.

Hilbert space: orthonormal bases (existence, countable if and only if separable), orthogonal complements, Hilbert space direct sums, bounded linear operators on a Hilbert space as a C*-algebra. Completely bounded and completely positive operators.

Applications Fourier series in $L_2[0,2\pi]$, Wavelet bases for $L^2(\mathbb{R})$.
Objectives: This course aims to introduce general techniques used widely in analysis (and other branches of mathematics) and to treat a few topics that are active areas of research.

June 10, 1998