School of Mathematics

Module MA3423 — Topics in complex analysis I 2011-12
(JS & SS Mathematics, JS & SS Two-subject Moderatorship)

Lecturer: Dr. Dmitri Zaitsev

Requirements/prerequisites: prerequisite: 2325

Duration: Michaelmas term, 10 weeks

Number of lectures per week: 3 lectures including tutorials per week

Assessment:

ECTS credits:

End-of-year Examination: This module will be examined jointly with MA3424 in a 3-hour examination in Trinity term, except that those taking just one of the two modules will have a 2 hour examination.

Description:

Further detailed information about the course: `http://www.maths.tcd.ie/~zaitsev/342-2011-12/342.html`

Real and complex differentiability. Holomorphic functions. Branches of multi-valued functions. Branches of logarithm and of the nth root. Conformal mappings.

Complex integration along piecewise smooth paths. Antiderivatives. Calculating integrals using antiderivatives. Cauchy's theorem: Goursat's version for a triangle, for star-shaped regions and their unions, homotopy version. Elements of homology and homological version of Cauchy's theorem.

Cauchy's integral formula. Power series expansion of holomoprhic functions. Mean value property. Maximum modulus principle. Radius and disk of convergence of power series. Cauchy-Hadamard formula. Theorem of Morera. Cauchy's estimates. Liouville's theorem. Application to the Fundamental Theorem of Algebra. Compact convergence and Weierstrass theorem.

Order of zeroes. The identity principle. Laurent series expansion in a ring. Isolated singularities. Removable singularities, poles, essential singularities. Riemann extension theorem. Meromorphic functions. Casorati-Weierstrass theorem.

The argument principle. Rouché's theorem. Open mapping theorem. The univalence theorem (local injectivity criterion). Inverse function theorem.

Spaces of holomorphic functions. Seminorms. Montel's theorem. Biholomphic maps between open sets. The Riemann mapping theorem.

Möbius transformations. Riemann sphere (extended complex plane). Stereographic projection. Rationality of meromorphic functions on the Riemann sphere. Automorphisms of the Riemann sphere and the complex plane. Schwarz Lemma. Automorphisms of the disk. Cayley transform. Automorphisms of the upper half-plane. Homogeneity of the Riemann sphere, complex plane and disk.

Schwarz Reflection Principle. Mittag-Leffler's theorem.

Textbooks:

1. L. V. Ahlfors, Complex Analysis, Third Edition, McGraw-Hill, New York, 1978.
2. J. B. Conway, Functions of One Complex Variable, Second Edition, Graduate Texts in Mathematics 11, Springer-Verlag, New York, 1978.
3. R. Remmert, Theory of Complex Functions, Graduate Texts in Mathematics 122, Springer-Verlag, New York, 1991.
4. R. V. Churchill, J. W. Brown, Complex Variables and Applications, Fourth edition. McGraw-Hill Book Co., New York, 1984.
5. B. P. Palka, An Introduction to Complex Function Theory, Undergraduate Texts in Mathematics. Springer-Verlag, New York, 1991.

Learning Outcomes: On sucessful completion of this module, students will be able to:

- operate with holomorphic functions and branches of multi-valued holomorphic functions
- give the appropriate definitions, statement and proofs of Cauchy theorem and its consequences
- demonstrate the use of Morera and Rieman Extension theorems
- give examples of power and Laurent series and of isolated singularities that are removable, poles and essential

November 2, 2011