

School of Mathematics

Module MA2317 — Introduction to number theory
(SF & JS Mathematics, JS & SS Two-subject Moderatorship)

2009-10

Lecturer: Dr. Timothy Murphy

Requirements/prerequisites: There are no pre-requisites beyond school mathematics.

Duration: Michaelmas term, 11 weeks

Number of lectures per week: 3 lectures including tutorials per week

Assessment:

End-of-year Examination: 2 hour examination in Trinity term.

Description:

According to Gauss, “Mathematics is the queen of the sciences and number theory is the queen of mathematics.”

Number theory can be divided, very roughly, into 3 parts:

- Elementary number theory
- Algebraic number theory
- Analytic number theory

This course is concerned, basically, with elementary number theory, although we shall make a foray into the simplest topic in algebraic number theory, namely quadratic number fields. We shall also mention, without proof, the two basic results of analytic number theory, namely the Prime Number Theorem and Dirichlet’s Theorem on primes in arithmetic sequences.

We begin with the Fundamental Theorem of Arithmetic, Euclid’s Theorem that every natural number $n > 0$ is uniquely expressible as a product of primes. (This result is so familiar that one can easily overlook the subtlety of the proof, and the enormous step taken by Euclid or his school in establishing it.)

Elementary number theory is, to a large extent, the study of prime numbers. As a kind of game to go with the course, we shall join the hunt for the next largest prime number. This is certain to be a Mersenne prime, due to the Lucas-Lehmer test which can tell us whether enormously large Mersenne numbers $2^p - 1$ (where p is a prime) are prime or not. We shall be able to establish, as an exercise in quadratic number fields, the validity of this test.

On-line notes for the course will be available in <http://www.maths.tcd.ie/pub/Maths/Courseware/NumberTheory>.

In particular, the topics to be covered in the course may be found in the file *Contents.pdf* in that folder.

February 26, 2010