School of Mathematics

Module CS4002 — Category theory 2009-10
(JS & SS Mathematics, JS & SS Two-subject Moderatorship)

Lecturer: Dr. Arthur Hughes (Computer Science)

Requirements/prerequisites:

Duration: Hilary term, 11 weeks

Number of lectures per week: 3 lectures including tutorials per week

Assessment:

End-of-year Examination: 2 hour examination in Trinity term.

Description: (Preliminary.)

- Categories – functions of sets, definition of a category, examples of categories, isomorphisms, constructions on categories, free categories, foundations: large, small, and locally small.
- Abstract structures – epis and monos, initial and terminal ob jects, generalized elements, sections and retractions, products, examples of products, categories with products, Hom-sets.
- Duality – the duality principle, coproducts, equalizers, coequalizers.
- Groups and categories – groups in a category, the category of groups, groups as categories, finitely presented categories.
- Limits and colimits – subob jects, pullbacks, properties of pullbacks, limits, preservation of limits, colimits.
- Exponentials – exponential in a category, cartesian closed categories, Heyting algebras, equational definition, -calculus.
- Functors and naturality – category of categories, representable structure, stone duality, naturality, examples of natural transformations, exponentials of categories, functor categories, equivalence of categories, examples of equivalence.
- Categories of diagrams – Set-valued functor categories, the Yoneda embedding, the Yoneda Lemma, applications of the Yoneda Lemma, Limits in categories of diagrams, colimits in categories of diagrams, exponentials in categories of diagrams, Topoi.
- Adjoints – preliminary definition, Hom-set definition, examples of adjoints, order adjoints, quantifiers as adjoints, RAPL, locally cartesian closed categories, adjoint functor theorem.
- Monads and algebras – the triangle identities, monads and adjoints, algebras for a monad, comonads and coalgebras, algebras for endofunctors.

Bibliography: Awodey, S. (2006). Category Theory. Oxford Logic Guides 49, Oxford University Press.

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