

## School of Mathematics

### Course 419 — Partial Differential Equations

2008-09

(JS &amp; SS Mathematics

SS Theoretical Physics )

**Lecturer:** Dr. John Stalker

**Requirements/prerequisites:** 216

**Duration:** 19 weeks

**Number of lectures per week:** 3

**Assessment:** Assignments will count for 10% of the mark for 419.

**End-of-year Examination:** 3-hour annual examination in May/June (will count for 90% of the mark).

#### Description:

More or less everything worth describing can be described by partial differential equations. While there is not much useful that one can say about partial differential equations in general, one can say a fair amount about broad classes of partial differential equations, and a great deal about some particular partial differential equations. Luckily these include the differential equation most relevant to physics.

The course will cover standard introductory material from the theory of partial differential equations. The approach will be fairly standard, except that I will emphasise the role of symmetry groups somewhat more than is usual.

The course will more or less follow the outline below. Please look at <http://www.maths.tcd.ie/~stalker/419> for further details.

- Introduction
  - Examples
  - Terminology
    - \* Order of an Equation
    - \* Ordinary vs. Partial Differential Equations
    - \* Linear vs. Non-linear Equations
    - \* Scalar Equations vs. Systems
  - First Order Scalar Equations
    - \* Method of Characteristics
    - \* Examples
    - \* Problems of Existence and Uniqueness
- The Wave Equation in 1+1 Dimensions
  - Method of Characteristics

- \* Explicit Solution
  - \* Existence
  - Causality and Energy
    - \* Domain of Dependence
    - \* Domain of Influence
    - \* Uniqueness
    - \* Stability
  - Weak Solutions
- Diffusion in 1+1 dimensions
  - Maximum Principle
    - \* Uniqueness
    - \* Stability
  - Symmetries
  - Fundamental Solution
    - \* Existence
- Boundary Problems and Reflection
  - Problems on the Half Line
    - \* Diffusion
      - Dirichlet Condition; Neumann Condition
    - \* Wave Equation
      - Dirichlet Condition, Neumann Condition
  - Problems on Finite Intervals
    - \* Dirichlet at Both Ends
    - \* Neumann at Both Ends
    - \* Dirichlet and Neumann
- Inhomogeneous Problems
  - Diffusion
  - Wave Equation
- Separation of Variables
  - Harmonic Oscillator on the Real Line
  - Diffusion on a Finite Interval
  - Wave Equation on a Finite Interval
  - Diffusion on the Real Line

- Wave Equation on the Real Line
- Harmonic Functions
  - Laplace Equation in 2 Dimensions
  - Maximum Principle
    - \* Uniqueness
    - \* Stability
  - Rectangular Domain
  - Circular Domain
  - Conformal Symmetry Group
  - Laplace Equation in 3 Dimensions
- Wave Equation in Higher Dimensions
  - 1+3 dimensions
  - 1+2 dimensions
- Burger's Equation
  - Solution by Method of Characteristics
  - Shocks
- Korteweg-de Vries
  - Solitons
  - Conservation Laws

**Textbooks:** The text for the course is Partial Differential Equations, An Introduction by Walter Strauss. You don't need to buy it immediately, but you should do so eventually.

December 10, 2008