School of Mathematics

Course 381 — Mathematical Economics 2008–09
(J.S./S.S. Mathematics, J.S./S.S. TSM)

Lecturer: Dr J. Thijssen & Dr. E. Denny

Requirements/prerequisites:

Duration: 24 weeks

Number of lectures per week: 2 lectures + 1 tutorial per week.

Assessment: One hand-in exercise for each semester, each counting for 10% towards the final grade.

End-of-year Examination: Three-hour annual examination will count for the remaining 80% of the final mark.

Description: The module will have two semester long parts, one on *Microeconomics* and the second on *the mathematics of financial instruments.*

Semester 1 (Dr. Thijssen)

For more information, see `http://www.tcd.ie/Economics/staff/thijssej`
Introduction

Modern microeconomic theory is highly mathematical and axiomatic in nature and, therefore, lends itself extremely well for study by mathematicians as an interesting application of fields such as analysis, topology, probability theory and differential equations. This module will introduce the corner stones of modern microeconomic theory, mainly by using tools from convex analysis. The three main subjects to be discussed are static optimisation, consumer and producer theory, and general equilibrium theory.

In the part on static optimisation, we will discuss techniques to solve different kinds of optimisation problems that arise in economics. In particular we will discuss optimisation of functions on Eucliadian spaces, optimisation of functions with equality constraints, and optimisation of functions with inequality constraints. The tools for these techniques have been studied already in Analysis I and Analysis II.

In the second part we will build axiomatic models of a fundamental agent in an economy: the consumer. We will see that the axioms are such that the objectives of these agents can be formulated mathematically as optimisation problems under constraints. Such problems will be studied in some details and the influence of the economic environment on agentsecisions will be studied (comparative statics). Furthermore, some simple economic policies will be analysed in relation to their effects on agentsell-being.

Finally, in the third part we will move from studying the individual to studying how these individuals interact. For simplicity we will not study production here, but only so-called exchange economies. The main problems to be discussed are the role of prices in an economy, the existence of prices equilibrating supply and demand of goods, and welfare properties of such equilibria.

Learning Outcomes After this module you should understand:

- The main techniques of static optimisation
- The concept of shadow prices
- The axioms underlying consumer and producer theory
- The derivation of the utility and profit maximisation problems from the axioms
- The duality between utility maximisation and expenditure minimisation
- The basic model of an exchange economy
- The basic general equilibrium proof
- The basic welfare properties of general equilibrium

After this module you should be able to:

- Solve standard optimisation problems
- Solve standard utility and profit maximisation problems
- Apply Slutsky's theorem to analyse welfare effects
- Solve for general equilibirum in simple exchange economies
- Solve for Pareto optimal allocations in simple exchange economies
- Find the core of simple exchange economies

After this module you should have a critical attitude to:

- The axiomatic theory of economics

Detailed Outline

I: Static Optimisation

- Optimisation of functions on $\mathbb{R}^n$.
- Optimisation of functions with equality constraints
- Optimisation of functions with inequality constraints

II: Consumer Theory

III: General Equilibrium in Exchange economies

- Exchange Economies
- Existence of Walrasian Equilibrium
- Welfare properties of Walrasian equilibria
- Core and Walrasian equilibirum

Semester 2 (Dr. Denny)

Aims: This module enables students to develop an in-depth understanding of the mathematics of financial instruments and the economic applications of stochastic calculus.

Introduction

This course provides a detailed study of the financial, probabilistic, and statistical frameworks essential to understanding the modern theory and practice of asset pricing. The course is divided into three main sections.

Part I will consist of an introduction to assets and derivative instruments, their characteristics, uses, and core principles. As a key element in valuing assets, the structure of interest-rates will also be covered in Part I. Fundamental financial concepts will be developed and the no-arbitrage principles will be emphasised. Mean variance theory will be introduced and the CAPM and arbitrage models for asset pricing will be developed in detail.

Part II of the course will be concerned with the pricing of derivative products. The basic principles and context of discrete and continuous time market theory will underpin this section of the course, culminating in the pricing of various kinds of vanilla and exotic options. Binomial trees using arbitrage arguments and risk-neutral valuation ideas for the pricing of options will be covered. The remaining topics will be drawn from: an overview of the Brownian Motion process, including an elementary treatment of stochastic differential equations; modelling stock prices using geometric Brownian Motion; Ito's lemma; and the Black-Scholes partial differential equation.

The final section will concentrate on martingales. Martingales are zero-drift stochastic processes and this part of the course will evaluate their uses in asset pricing and their relevance in stochastic modelling. Examples of martingales will be developed and a methodology for the pricing of martingales will be covered.

Learning Objectives: Upon completion of this course students should be able to:

- Comprehensively describe the characteristics and uses of a wide variety of financial derivatives
- Proof and application of CAPM and arbitrage theorem for the pricing of assets
- Demonstrate the precise mathematical detail of the definition and construction of the Ito integral and its uses
- Have a thorough grasp of Black-Scholes methodology, its PDE and martingale formulations, and its application in deriving option prices in continuous time models.

Outline:

I: Asset Pricing

- General introduction to assets, options, futures and other derivatives.
- Asset Pricing Models

II: Derivative valuation

- An introduction to derivative instruments and their valuation

- Continuous time models
- Option Pricing

III: Martingales

December 19, 2008