

School of Mathematics

Course 321 - Functional Analysis

2008-09

(Optional JS & SS Mathematics, SS Two-subject Moderatorship)

Lecturer: Prof. R. Timoney

Requirements/prerequisites: The material in 221 is going to be relied upon.

Duration: 19 weeks

Number of lectures per week: 3

Assessment: Regular assignments.

End-of-year Examination: One 3-hour examination

Description:

The following is a draft syllabus.

Banach spaces: definitions and examples ($C(X)$, ℓ^p , Hölder and Minkowski inequalities, closed subspaces, c_0 , $L^p(\mathbb{R})$, $L^p[0, 1]$).

Completeness for metric spaces: completion. Baire category.

Linear operators: examples of continuous inclusions among ℓ^p and $L^p[0, 1]$ spaces, n -dimensional normed spaces isomorphic. Open mapping and closed graph theorems. Uniform boundedness principle.

Fundamental Concepts: Partial order, Zorn's lemma as an axiom, application to bases of vector spaces; cardinal numbers; ordinal numbers.

Dual spaces: Hahn-Banach theorem, canonical isometric embedding in double dual, reflexivity.

Hilbert space: orthonormal bases (existence, countable if and only if separable), orthogonal complements, Hilbert space direct sums, bounded linear operators on a Hilbert space as a C^* -algebra. Completely bounded and completely positive operators.

Applications: Fourier series in $L^2[0, 2\pi]$.

There is a web site <http://www.maths.tcd.ie/~richardt/321> for the course.

Objectives: This course aims to introduce general techniques used widely in analysis (and other branches of mathematics) and to treat a few topics that are active areas of research.

October 5, 2008