School of Mathematics

Course 113 — Linear Algebra 2008-09
(JF Mathematics
JF Theoretical Physics
JF TSM Mathematics)

Lecturer: Dr. Vladimir Dotsenko

Requirements/prerequisites: None.

Duration: 24 weeks

Number of lectures per week: 3

Assessment:

End-of-year Examination: 3-hour examination

Description:

Further information about the course can be found at `http://www.maths.tcd.ie/~vdots/index113-0809.html`

The purpose of this course is to give you an introduction to Linear Algebra, which is one of the most important technical tools both in Pure and Applied Maths. The topics covered will be

1. Linear algebra in 2d and 3d. Vectors. Dot product. Cross product. Applications to geometry.
2. Systems of simultaneous linear equations. Examples.
3. Gauss–Jordan elimination.
4. (Reduced) row echelon form for a rectangular matrix.
5. Fredholm's alternative. Applications.
6. Computing the inverse matrix using row operations.
7. Odd and even permutations. Determinants.
8. Row and column operations on determinants. Determinant of the transpose matrix.
9. Minors. Cofactors. Adjoint matrix. Computing the inverse matrix using determinants.
10. Cramer's rule for systems with the same number of equations and unknowns.
11. Coordinate vector space. Ranks. Maximal size of nonzero minors is equal to the rank.
12. Fields: rationals, reals, and complex.
13. Abstract vector spaces.

14. Linear independence and completeness. Exchange lemma.

15. Bases and dimensions. Subspaces.

16. Linear operators. Matrices.

17. Change of basis. Transition matrices. Similar matrices define the same linear operator in different bases.

18. Characteristic polynomials. Eigenvalues and eigenvectors.

19. Diagonalisation in the case when the characteristic polynomial has no multiple roots.

20. Cayley-Hamilton theorem. Minimal polynomial of a linear operator. Examples (operators with $A^2 = A$).

21. Invariant subspaces. Direct sums.

22. Normal form for a nilpotent operator. Jordan normal form (Jordan Decomposition Theorem).

23. Applications: computing functions of matrices, solving differential equations, finding closed expressions for recursively defined sequences.

24. Orthonormal bases; Gram-Schmidt orthogonalisation.

25. Orthogonal complements and orthogonal direct sums. Bessel's inequality.

26. Bilinear and quadratic forms. Sylvester's criterion. The law of inertia.

27. Spectral Theorem for symmetric operators.

28. Complex Hilbert spaces.

29. Two commuting linear operators have a common eigenvector.

30. Spectral Theorem for normal operators.

September 15, 2008