School of Mathematics

Course 221 — Analysis 2007-08
(SF Mathematics, SF Two-subject Moderatorship with Economics, optional JS Two-subject Moderatorship)

Lecturer: Dr. D.R. Wilkins and Dr. F. Jaeck

Requirements/prerequisites:

Duration: 24 weeks

Number of lectures per week: 3

Assessment: several assignments, providing 10% of the credit for the course

End-of-year Examination: One 3-hour examination

Description: See `http://www.maths.tcd.ie/~dwilkins/Courses/221/` for more detailed information.

First semester:

Section 1: Basic Theorems of Real Analysis. The Least Upper Bound Principle; convergence of bounded monotonic sequences of real numbers; upper and lower limits; Cauchy's Criterion for Convergence; the Bolzano-Weierstrass Theorem; the Intermediate Value Theorem.

Section 2: Analysis in Euclidean Spaces. Euclidean spaces; definition and basic properties of convergence and limits for sequences of points in Euclidean spaces; definition and basic properties of continuity for functions between subsets of Euclidean spaces; uniform convergence; open and closed sets in Euclidean spaces.

Section 3: Metric Spaces. Definition of a metric space; definition and basic properties of convergence and limits for sequences of points in a metric space; definition and basic properties of continuity for functions between metric spaces; open and closed sets in metric spaces; continuous functions and open and closed sets; homeomorphisms.

Section 4: Complete Metric Spaces, Normed Vector Spaces and Banach Spaces. Complete metric spaces; normed vector spaces; bounded linear transformations; spaces of bounded continuous functions; the Contraction Mapping Theorem; Picard's Theorem; the completion of a metric space.

Section 5: Topological Spaces. Topological spaces; Hausdorff spaces; subspace topologies; continuous functions between topological spaces; homeomorphisms; sequences and convergence; neighbourhoods, closures and interiors; product topologies; cut and paste constructions; identification maps and quotient topologies; connectedness.

Section 6: Compact Spaces. Definition and basic properties of compactness; compact metric spaces; the Lebesgue Lemma; uniform continuity; the equivalence of norms on a finite-dimensional vector space.

Second semester: measure theory; the Lebesgue integral.

October 3, 2007