School of Mathematics

Course 111 — Algebra 2000-2001
(JF Mathematics, Theoretical Physics & Two-subject Moderatorship)

Lecturer: Dr. C. Ó Dúnlaing

Requirements/prerequisites: None

Duration: 24 weeks

Number of lectures per week: 2 and 1 tutorial

Assessment: Continuous assessment through twenty homeworks and three short examinations spread over the academic year.

End-of-year Examination: No annual examination. Those who fail through continuous assessment must take a supplemental examination (in September)

Description:

Number theory: The natural number system and Peano's axioms
The integers , divisibility, and congruence modulo n
Remainder modulo n and integer division

Groups: Semigroups, monoids, and groups
Groups
Additive subgroups of $\mathbb{Z}$
The symmetric group S_n
Generators for S_n
Parity and the alternating group
Binary relations, equivalence relations, and partitions
Cosets, Lagrange's Theorem, and Fermat's Theorem
Normal subgroups and quotient groups
Greatest common divisor
Multiplicative group $\mathbb{Z}_n^*$
First isomorphism theorem for groups
Prime factorisation theorem
A Sylow theorem

Rings and fields: Rings
Zero divisors, integral domains, and fields.
Ring homomorphisms
Characteristic of a ring
Polynomials
Division algorithm for polynomials over a field
Factorising polynomials
Gauss's Lemma and Eisenstein's Criterion
Ring homomorphisms and ideals

Principal ideal domains

Solving equations: Dimension of extension fields
Ruler-and-compass constructions
Cubic equations
The Galois group of an extension field
Normal extensions, stable intermediate fields, and splitting fields.
Certain standardised radical extensions have solvable group; radical splitting fields have solvable group.
A polynomial equation not solvable by radicals

A last result: Finite multiplicative subgroups of a field
Textbooks: John R. Durbin, *Modern algebra – an introduction*, contains some but not all of the material.

June 14, 2004