School of Mathematics

Course 425 — Differential Geometry 2003–04
(Optional JS & SS Mathematics, SS Two-subject Moderatorship)

Lecturer: Dr. Dmitri Zaitsev

Requirements/prerequisites: Basic point set topology (covered in Course 212, see e.g. the lecture notes by David Wilkins `http://www.maths.tcd.ie/~dwilkins/Courses/212/`) and analysis in several real variables (covered in Course 221, see e.g. the lecture notes by David Simms `http://www.maths.tcd.ie/pub/coursework/211/` and the references below). Quick introduction to the necessary concepts will be given.

Duration: 21 weeks.

Number of lectures per week: 3

Assessment: Regular assignments

End-of-year Examination: One 3-hour examination

Description: (To be extended)

The course is thought as a "classical" course in Differential Geometry covering most of basic material traditionally associated with Differential and Riemannian Geometry. Geometrical structures on differentiable manifolds, going back to the mathematical formulation of classical mechanics, play a central role in modern mathematics and physics, in particular, in Riemannian geometry, Morse theory, Hodge theory, and the theory of partial differential operators.

Topics:

Regular plane curves and space curves. Curvature and torsion. Geometry of submanifolds of $\mathbb{R}^n$. Introduction to smooth manifolds. Tangent spaces. Vector fields. Lie groups. Sards theorem. Whitney embedding theorem. Differential forms. Integration on manifolds. Stokes theorem.

Riemannian geometry. Connections. Geometry of surfaces in $\mathbb{R}^3$. Geodesics. The variation of the length and energy functionals for smooth curves. Geodesic completeness and the theorems of Hopf and Rinow. Jacobi fields.

Additional information and feedback form can (or will) be found at
`http://www.maths.tcd.ie/~zaitsev/425.html`

Textbooks:

M.P. do Carmo, *Riemannian Geometry*, Birkhäuser, (1992).
W. Kühnel, *Differential geometry. Curves—surfaces—manifolds*, American Mathematical Society, Providence, RI, (2002).
M. Berger; B. Gostiaux, *Differential geometry: manifolds, curves, and surfaces*, Graduate Texts in Mathematics, 115. Springer-Verlag, New York, (1988).
M.W. Hirsch, *Differential topology.* Graduate Texts in Mathematics, 33. Springer-Verlag, New York, (1976) or (1994).
M. Spivak, *A Comprehensive Introduction to Differential Geometry*, Publish or Perish, (1979).
S. Kobayashi, K. Nomizu, *Foundations of differential geometry*, Interscience Publishers, (1963).

An excellent book for reviewing analysis and basic topology is
W. Rudin, *Principles of mathematical analysis*, McGraw-Hill Book Co., (1976).

November 3, 2003