

School of Mathematics

Course 111 — Algebra

2003-2004

(JF Mathematics, Theoretical Physics & Two-subject Moderatorship)

Lecturer: Dr. C. Ó Dúnlaing**Requirements/prerequisites:** None**Duration:** 24 weeks**Number of lectures per week:** 2 and 1 tutorial**Assessment:** Continuous assessment through twenty homeworks and three short examinations spread over the academic year.**End-of-year Examination:** No annual examination. Those who fail through continuous assessment must take a supplemental examination (in September)**Description:** The natural number system and Peano's axiomsThe integers, divisibility, and congruence modulo n Remainder modulo n and integer division

Semigroups, monoids, and groups

Groups

Additive subgroups of \mathbb{Z} The symmetric group S_n Generators for S_n

Parity and the alternating group

Binary relations, equivalence relations, and partitions

Cosets, Lagrange's Theorem, and Fermat's Theorem

Normal subgroups and quotient groups

Greatest common divisor

Multiplicative group \mathbb{Z}_n^*

First isomorphism theorem for groups

Prime factorisation theorem

A Sylow theorem

Rings

Zero divisors, integral domains, and fields.

Ring homomorphisms

Characteristic of a ring (omitted)

Polynomials

Division algorithm for polynomials over a field

Factorising polynomials

Gauss's Lemma and Eisenstein's Criterion

Ring homomorphisms and ideals

Principal ideal domains

Dimension of extension fields

Ruler-and-compass constructions

Cubic equations

The Galois group of an extension field

Normal field extensions

Stable intermediate fields

Splitting fields

Radical field extensions and solvability

A polynomial equation not solvable by radicals

Finite multiplicative subgroups of a field

Textbooks: John R. Durbin, *Modern algebra – an introduction*, contains some but not all of the material.

June 14, 2004