School of Mathematics

Course 211 — Linear algebra & Differential Forms 2002-03
(SF Mathematics, SF Theoretical Physics, SF Two-subject Moderatorship with Economics & JS Two-subject Moderatorship)

Lecturer: Prof. D.J. Simms

Requirements/prerequisites: 111, 131

Duration: 24 weeks

Number of lectures per week: 3

Assessment: Some assignments, which do not contribute to the final grade

End-of-year Examination: One 3-hour end of year examination

Description: Vector space over a field, coordinate functions with respect to a basis, dual space, M and M^* mutually dual, homogeneous linear equations, linear operators, matrix with respect to a basis, Einstein index notation, conjugacy classes, trace, characteristic polynomial, rank, direct sums, invariant subspaces, eigenspaces, diagonalisability, simultaneous diagonalising, Hamilton-Cayley, primary decomposition theorem, Jordan form, minimum polynomial, zeros of minimum polynomial, diagonalizability iff minimum polynomial is a product of distinct linear factors.

Bilinear forms, symmetric, hermitian, non-singular scalar product space, covariant and contravariant components, raising and lowering indices, $M = N \oplus N^{\perp}$ if N finite dim non-singular, classification of scalar products: complex symmetric, real symmetric, hermitian, diagonalisation, quadratic forms, Sylvester law, diagonalisation using determinants (Jacobi), condition for positive definite, Schwarz and triangle unequalities, adjoint operator, self-adjoint, isometry, normal operator, spectral theorem, orthogonal, unitary, Lorentz transformations, simultaneous reduction of quadratic forms.

Tensor addition, multiplication, contraction, construction of invariants, wedge product of skew-symmetric tensors, categories and functorial properties, volume element, Hodge star operator.

Tensor fields, metric tensor and line elements, gradient vector, tangent and normal, orientation, volume and area elements.

Exterior derivative, closed and exact forms, integration of forms.

n-dim vector analysis with wedge product and Hodge star operator, Poincare lemma, Gaussian curvature, Stokes's theorem and applications.

March 27, 2003