School of Mathematics

Course 131 — Mathematical Methods 2002-03
(JF Mathematics, Theoretical Physics & Two-Subject Moderatorship)

Lecturer: Prof. Petros Florides & Dr. Dmitri Zaitsev

Requirements/prerequisites: None

Duration: 24 weeks

Number of lectures per week: 3

Assessment:

End-of-year Examination: 3 hour paper in June

Description:
General introduction to vectors and linear vector spaces, vectors in 3-dimensions, application to 3-dimensional geometrical problems. Euclidean spaces, vector addition (triangle and parallelogram law), multiplication of a vector by a scalar, and the properties of these two operations which make the set of all vectors a linear vector space (over the reals). The set of all real n-tuples, as an example of a linear vector space.

Resolution of vectors along any two non-zero non-parallel vectors in dimension 2 and along any three non-zero non-coplanar vectors in dimension 3. The notion of linearly independent vectors and vector bases. Orthonormal bases, scalar and vector products, triple scalar and triple vector products and geometrical interpretations. Rotation of an orthonormal basis and the relationship between the old and the new components of a vector. A new definition of a (cartesian) vector based on this relationship. The Einstein summation and range conventions. The Kronecker delta and the Levi-Civita symbol and applications to vectors. Generalisation to vectors in dimension n.

Matrices; motivation and definition. Algebra of matrices and multiplication of two matrices. Determinants of square matrices, motivation, definition and main properties. Elementary row and column operations. Gaussian elimination algorithm. Cofactor expansion of determinants. Invertibility of matrices and the formula for the inverse. Solution of a system of linear equations, Cramer's rule. Eigenvalues and eigenvectors of matrices, diagonalization of matrices. Application to linear ordinary differential equations

Review of calculus in 1-dimension, introduction to partial differentiation, gradient operator and its geometrical significance. Taylor polynomials, Taylor series. Maxima and minima (extreme values), local and absolute. Critical points, 1st and 2nd derivative tests. Extreme values subject to constraints, Lagrange multipliers. Multiple and iterate integrals, line, surface and volume integrals, change of variable, Jacobians.

Additional information and feedback form can (or will) be found at
`http://www.maths.tcd.ie/~zaitsev/131.html`

References

1. G. H. Thomas Jr and R. L. Finney : Calculus and Analytic Geometry
2. D. E. Bourne and P. C. Kendall : Vector Analysis and Cartesian Tensors
3. B. Kolman : Introductory linear algebra with applications.
4. M. O'Nan : Linear algebra
5. L. W. Mansfield : Linear algebra with geometric applications
6. S. Lang : Undergraduate Analysis; Calculus of several variables
7. W. Rudin : Principles of mathematical analysis

April 27, 2003